6.1 Slope Fields

Find the general solution to the differential equation.

"derivative"

ex: $\frac{dy}{dt} = \cos t \, e^{\sin t}$

$$\int dy = \int \cos t \, e^{(\sin t)} \, dt$$

$$y = \int \cos t \cdot e^{u} \cdot \frac{du}{\cos t}$$

$$y = \int e^{u} \, du$$

$$y = e^{u} + C$$

$$\boxed{y = e^{\sin t} + C}$$

$u = \sin t$

$\frac{du}{dt} = \cos t$

$\frac{du}{\cos t} = dt$

Ex: $\frac{dy}{dx} = \frac{1}{1-2x} \cdot dx$

$\int dy = \int \frac{1}{(1-2x)} dx$

$u = 1-2x$

$y = \int \frac{1}{u} \cdot \frac{du}{-2}$

$\frac{du}{dx} = -2$

$y = -\frac{1}{2}\int \frac{1}{u} du$

$\frac{du}{-2} = dx$

$y = -\frac{1}{2}\ln|u| + C$

$\boxed{y = -\frac{1}{2}\ln|1-2x| + C}$ general solution

Find the particular solution to the differential equation

OR Solve the initial value problem explicitly.

Ex: $\frac{dy}{dx} = 2x - 3$ @ $(1,3)$ ← pt

$\int dy = \int (2x-3)\,dx$

$y = x^2 - 3x + C$ ← general solution

$3 = 1^2 - 3(1) + C$

$5 = C$

$\boxed{y = x^2 - 3x + 5}$ ← particular solution

ex: $\frac{dy}{dx} = \cos x - 3x^2$ $y=5$ when $x=0$

$$\int dy = \int (\cos x - 3x^2)\, dx$$

$$y = \sin x - x^3 + C \rightarrow \text{general solution}$$

$$5 = \sin 0 - 0^3 + C$$

$$5 = C$$

$$\boxed{y = \sin x - x^3 + 5}$$ particular solution

ex: $\frac{dA}{dt} = 10t^9 + 5t^4 - 2t + 4$ $A=6$ when $t=1$

$$\int dA = \int (10t^9 + 5t^4 - 2t + 4)\, dt$$

$$A = t^{10} + t^5 - t^2 + 4t + C \leftarrow \text{general solution}$$

$$6 = 1^{10} + 1^5 - 1^2 + 4(1) + C$$

$$6 = 1 + 1 - 1 + 4 + C$$

$$1 = C$$

$$\boxed{A = t^{10} + t^5 - t^2 + 4t + 1}$$ particular solution