

6.1 Vectors in the Plane

Target 8A: Perform vector operations: scalar multiple and sums and represent them graphically

Target 8B: Perform vector operations: magnitude, direction angle, and unit vector

Review of Prior Concepts

$$P = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, Q = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, R = \begin{bmatrix} -4 \\ 2 \end{bmatrix}, S = \begin{bmatrix} -1 & 2 \\ 6 & 1 \end{bmatrix}.$$

Simplify

a) $P + Q$

$$P + Q = \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

b) $Q + R$

$$Q + R = \begin{bmatrix} 4 \\ -2 \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

c) $Q - R$

$$Q - R = \begin{bmatrix} 4 \\ -2 \end{bmatrix} - \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

d) $3Q$

$$3Q = 3 \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \end{bmatrix}$$

e) QS

QS cannot be done
b/c cannot multiply
(2×1)(2×2) matrices

f) SR

$$SR = \begin{bmatrix} -1 & 2 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1(-4) + 2(2) \\ 6(-4) + 1(2) \end{bmatrix} = \begin{bmatrix} 8 \\ -22 \end{bmatrix}$$

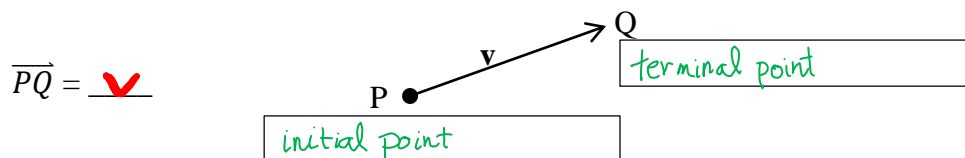
More Practice

Operations with Matrices

<http://www.mathsisfun.com/algebra/matrix-introduction.html>
<https://www.khanacademy.org/math/precalculus/precalc-matrices#adding-and-subtracting-matrices>
http://www.algebra-lab.org/lessons/lesson.aspx?file=algebra_matrix_operations.xml
<https://www.youtube.com/watch?v=xr6qsiEznKU>
<https://www.youtube.com/watch?v=SPFWVUkxk8E>
https://www.youtube.com/watch?v=kuixY2bCc_0
<https://www.youtube.com/watch?v=sYIOjyPyX3g>

Vectors

Vector -- a directed line segment _____ with an initial point and terminal point.
The vector is represented with a **bold** lowercase letter.



Note: The arrow on the vector shows the direction --- a vector is NOT a ray
The vector above stops at Q. ← (is a segment)

A vector has magnitude (length of the vector) represented by $|\overrightarrow{PQ}|$ or $|\mathbf{v}|$
and direction (slope of the vector) represented by an angle.

Vectors are equal if both magnitude AND direction are equal.

Example

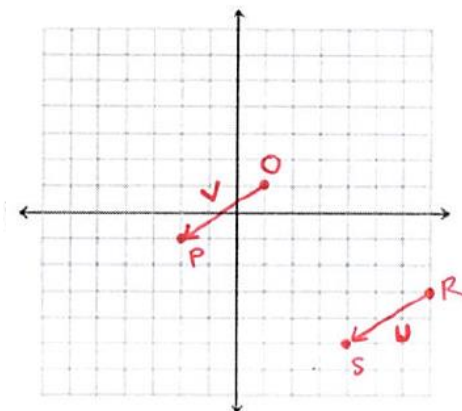
\mathbf{u} is represented by \overrightarrow{RS} and \mathbf{v} is represented by \overrightarrow{OP} , where $R = (7, -3)$, $S = (4, -5)$, $O = (1, 1)$, $P = (-2, -1)$. Prove $\mathbf{u} = \mathbf{v}$.

magnitude of $\overrightarrow{RS} = |\overrightarrow{RS}| = \sqrt{(7-4)^2 + (-3-(-5))^2}$
 $|\mathbf{u}| = \sqrt{13}$

magnitude of $\overrightarrow{OP} = |\overrightarrow{OP}| = \sqrt{(1-(-2))^2 + (1-(-1))^2}$
 $|\mathbf{v}| = \sqrt{13}$

slope of $\overrightarrow{RS} = \frac{2}{3}$
 slope of $\overrightarrow{OP} = \frac{2}{3}$ } = \mathbf{v}

$\therefore \mathbf{u} = \mathbf{v}$



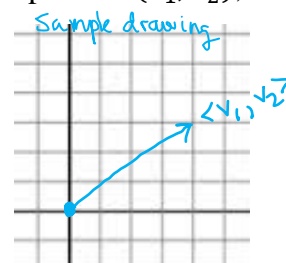
Component Form and Magnitude of a Vector

If \mathbf{v} is a vector in a plane equal to the vector with initial point at $(0,0)$ and terminal point at (v_1, v_2) , then the **component form** of \mathbf{v} is

$$\mathbf{v} = \langle v_1, v_2 \rangle$$

The *components* are v_1 and v_2 .

The vector is called the *position vector*, where position = $\langle v_1, v_2 \rangle$



Writing a Vector in Component Form

Given $\overrightarrow{PQ} = \mathbf{v}$, where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, then

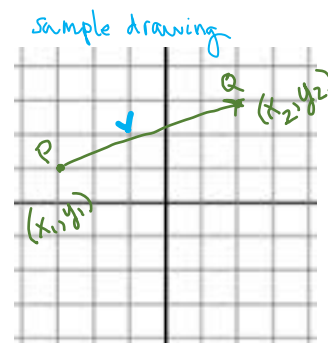
$$\mathbf{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

$$= \langle v_1, v_2 \rangle$$

terminal pt - initial pt

Magnitude of $\mathbf{v} = |\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 or $\sqrt{v_1^2 + v_2^2}$

distance formula
 Pythagorean theorem



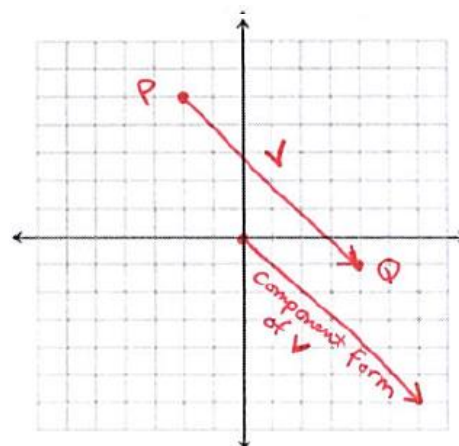
Example

Write the vector, $\mathbf{v} = \overrightarrow{PQ}$, in component form and find the magnitude of \mathbf{v} , where $P = (-2, 5)$ and $Q = (4, -1)$.

$$\mathbf{v} = \langle 4 - (-2), -1 - 5 \rangle$$

$$\mathbf{v} = \langle 6, -6 \rangle$$

magnitude = $\sqrt{(4 - (-2))^2 + (-1 - 5)^2}$
 $= \sqrt{(6)^2 + (-6)^2}$
 $= \sqrt{72}$
 or $6\sqrt{2}$



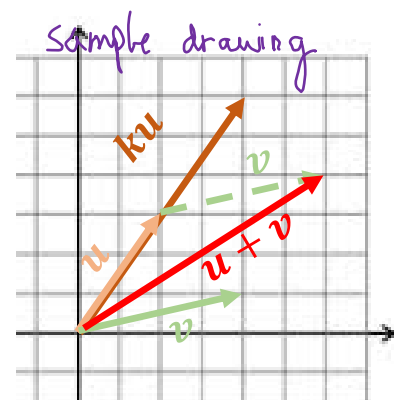
Vector Addition and Scalar Multiplication

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ and k is a constant, then

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

and

$$k\mathbf{u} = \langle ku_1, ku_2 \rangle$$



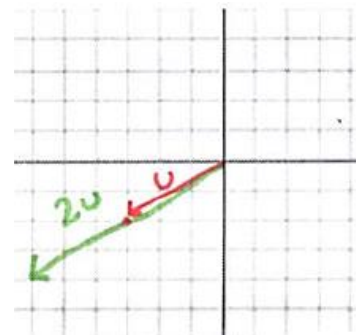
Example

Let $\mathbf{u} = \langle -3, -2 \rangle$ and $\mathbf{v} = \langle 1, 4 \rangle$. Find:

a) $2\mathbf{u}$

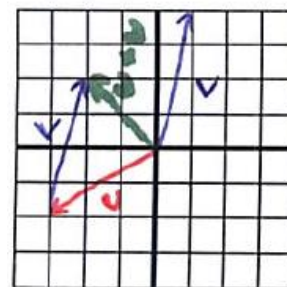
$$2\mathbf{u} = 2\langle -3, -2 \rangle \\ = \langle -6, -4 \rangle$$

a)



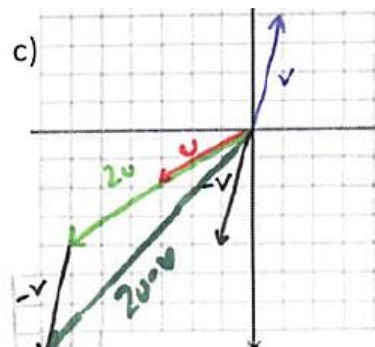
b) $\mathbf{u} + \mathbf{v} = \langle -3, -2 \rangle + \langle 1, 4 \rangle \\ = \langle -2, 2 \rangle$

b)



c) $2\mathbf{u} - \mathbf{v} = 2\langle -3, -2 \rangle - \langle 1, 4 \rangle \\ = \langle -6, -4 \rangle - \langle 1, 4 \rangle \\ = \langle -7, -8 \rangle$

c)



Vector Practice

Component Form and Magnitude

Find the component form and magnitude of each vector.

$$v = \langle 4 - (-5), 6 - 2 \rangle$$

$$= \langle 9, 4 \rangle$$

$$|v| = \sqrt{9^2 + 4^2}$$

$$= \sqrt{97}$$

$$u = \langle 3 - 5, -5 - (-1) \rangle$$

$$= \langle -2, -4 \rangle$$

$$|u| = \sqrt{(-2)^2 + (-4)^2}$$

$$= \sqrt{20} \text{ or } 2\sqrt{5}$$

$$a = \langle -1 - (-4), -5 - (-1) \rangle$$

$$= \langle 3, -6 \rangle$$

$$|a| = \sqrt{3^2 + (-6)^2} = \sqrt{45} \text{ or } 3\sqrt{5}$$

$$v = \langle 9, 4 \rangle$$

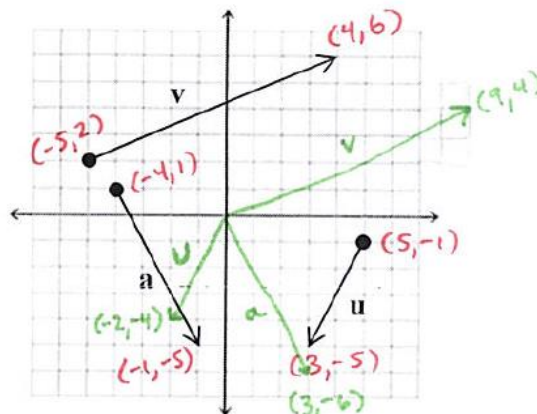
$$|v| = \sqrt{97}$$

$$u = \langle -2, -4 \rangle$$

$$|u| = 2\sqrt{5}$$

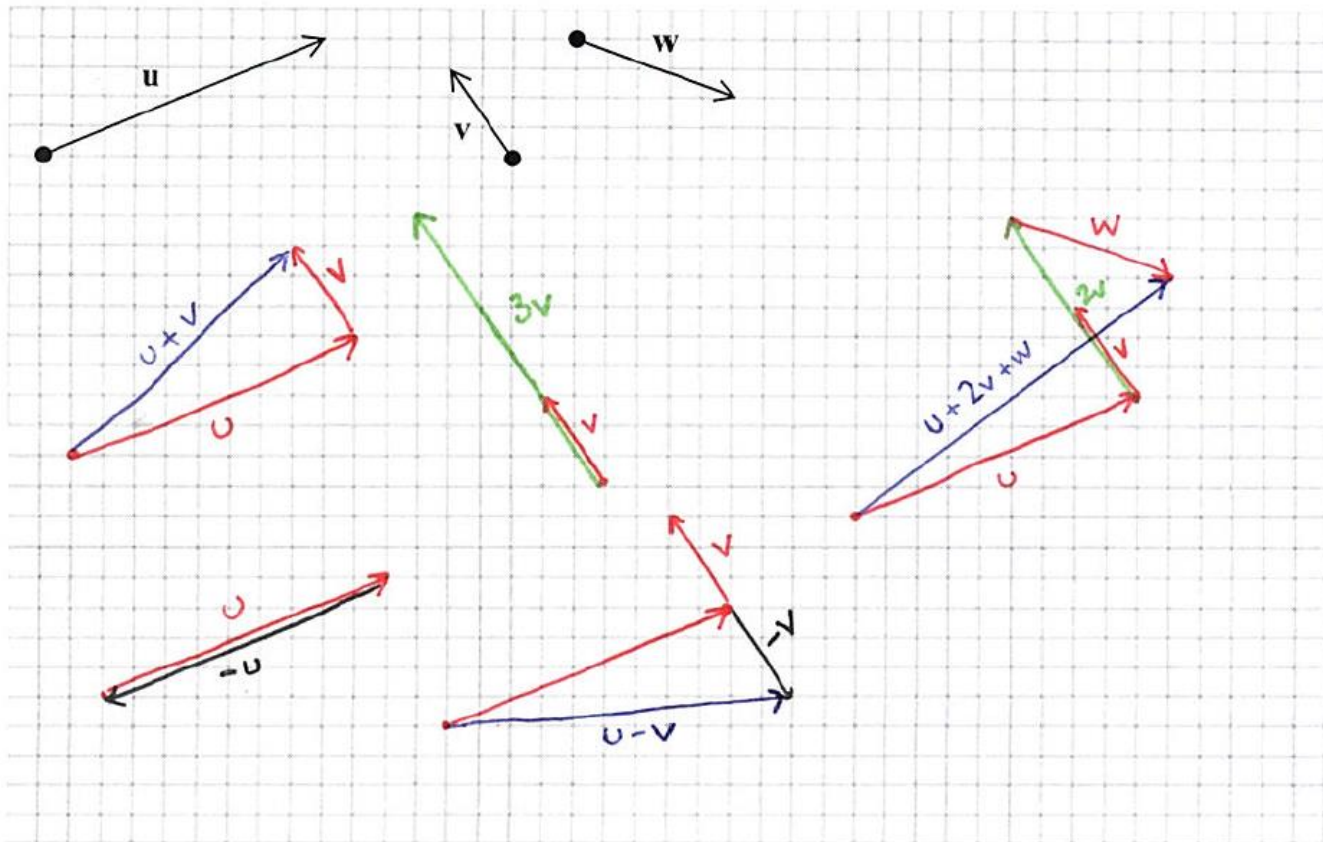
$$a = \langle 3, -6 \rangle$$

$$|a| = 3\sqrt{5}$$



Addition and Scalar Multiplication

Sketch the resultant vectors: $u + v$, $3v$, $-u$, $u - v$, $u + 2v + w$



More Practice**Introduction to Vectors**

<https://www.mathsisfun.com/algebra/vectors.html>

<https://www.khanacademy.org/math/precalculus/vectors-precac>

<http://philschatz.com/precalculus-book/contents/m49412.html>

<https://www.varsitytutors.com/precalculus-help/matrices-and-vectors/geometric-vectors>

https://youtu.be/_YkIivLaVJs

<https://youtu.be/60btq9PN8IM>

<https://youtu.be/IKzR0Odurm0>

Component Form of Vectors

<https://braingenie.ck12.org/skills/108146>

<https://www.khanacademy.org/math/precalculus/vectors-precac/vector-basic/v/example-finding-components-of-a-vector>

<https://www.mathway.com/examples/precalculus/vectors/finding-the-position-vector?id=582>

<https://youtu.be/pimr9I92GZY>

<https://youtu.be/-0qEDcZZS9E>

<https://youtu.be/wz5AvyKWjb8>

Vector Addition and Scalar Multiplication

<http://www.mathplanet.com/education/geometry/transformations/vectors>

<http://www.softschools.com/math/pre-calculus/scalar-multiplication-and-vector-addition/>

<https://www.khanacademy.org/math/precalculus/vectors-precac#scalar-multiplication>

<https://youtu.be/pNMrYACjHXQ>

<https://youtu.be/Z1o-DGohM60>

https://youtu.be/_YkIivLaVJs

<https://youtu.be/lulSApFPwIM>

<https://youtu.be/0i9Cp-6QuUk>

Homework Assignment

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