For each problem, find the particular solution of the differential equation.

1. $f'(x) = x^2$, $f(0) = 1$

$\int f'(x) = \int x^2\,dx$

$f(x) = \frac{1}{3}x^3 + C$

$f(0) = \frac{1}{3}(0)^3 + C$

$1 = C$

$f(x) = \frac{1}{3}x^3 + 1$

2. $f'(x) = -\sin x$, $f(\pi) = 3$

$\int f'(x) = \int -\sin x\,dx$

$f(x) = \cos x + C$

$f(\pi) = \cos\pi + C$

$3 = -1 + C$

$4 = C$

$f(x) = \cos x + 4$

3. $f''(x) = x^2$, $f'(0) = 6$, $f(0) = 3$

$\int f''(x) = \int x^2\,dx$

$f'(x) = \frac{1}{3}x^3 + C$

$f'(0) = \frac{1}{3}(0)^3 + C$

$6 = C$

$f'(x) = \frac{1}{3}x^3 + 6$

$\int f'(x) = \int(\frac{1}{3}x^3 + 6)\,dx$

$f(x) = \frac{1}{12}x^4 + 6x + C$

$f(0) = \frac{1}{12}(0)^4 + 6(0) + C$

$3 = C$

$f(x) = \frac{1}{12}x^4 + 6x + 3$

4. The rate of growth $\frac{dP}{dt}$ of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days ($0 \le t \le 10$). That is,

$$\frac{dP}{dt} = k\sqrt{t}\,.$$

The initial size of the population is 500. After 1 day the population has grown to 600. Estimate the population after 7 days.

$dP = k\sqrt{t}\,dt$ $\qquad P(0) = 500,\ P(1) = 600,\ P(7) = ?$

$\int dP = k\int t^{1/2}\,dt$

$P(t) = k(\frac{2}{3}t^{3/2}) + C$

$P(0) = \frac{2k}{3}(0)^{3/2} + C$

$500 = C$

$P(t) = \frac{2k}{3}t^{3/2} + 500$

$P(1) = \frac{2k}{3}(1)^{3/2} + 500$

$600 = \frac{2k}{3} + 500$

$100 = \frac{2k}{3}$

$\frac{300}{2} = k$

$150 = k$

$P(t) = 100t^{3/2} + 500$

$P(7) = 100(7)^{3/2} + 500$

$= 2352$ bacteria