DATE: ________________

Euler's Method AP Practice

1. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.
Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

$\Delta x = \frac{\frac{1}{2}-0}{2} = \frac{1}{4}$

(x,y)	$\frac{dy}{dx}$	$\underbrace{\frac{dy}{dx}(\Delta x)}_{\Delta y}$	$\Delta y + y$
$(0,-1)$	$(-1)^2(2(0)+2)$ 2	$2\left(\frac{1}{4}\right) = \frac{1}{2}$	$\frac{1}{2} + -1 = -\frac{1}{2}$
$\left(\frac{1}{4}, -\frac{1}{2}\right)$	$\left(-\frac{1}{2}\right)^2\left(2\left(\frac{1}{4}\right)+2\right)$ $\frac{5}{8}$	$\frac{5}{8}\left(\frac{1}{4}\right) = \frac{5}{32}$	$\frac{5}{32} + -\frac{1}{2} = \frac{5}{32} - \frac{16}{32} = -\frac{11}{32}$
$\left(\frac{1}{2}, \frac{-11}{32}\right)$			

$\boxed{f\left(\frac{1}{2}\right) = \frac{-11}{32}}$

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2. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f', the derivative of f, are given for selected values of x in the table below.

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

$\Delta x = \frac{1.4-1}{2} = .2$

(x,y)	$\frac{dy}{dx}$	$\underbrace{\frac{dy}{dx}(\Delta x)}_{\Delta y}$	$\Delta y + y$
$(1,15)$	8	$8(.2) = 1.6$	$1.6 + 15 = 16.6$
$(1.2, 16.6)$	12	$12(.2) = 2.4$	$2.4 + 16.6 = 19$
$(1.4, 19)$			

$\boxed{f(1.4) = 19}$

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3. Consider the differential equation $\frac{dy}{dx} = 1 - y$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 0$. For this particular solution, $f(x) < 1$ for all values of x.
Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

$\Delta x = \frac{0-1}{2} = -\frac{1}{2}$

$f(1) = 0$

$f(\frac{1}{2}) = 0 + \left.\frac{dy}{dx}\right|_{(1,0)} (\Delta x)$

$= 0 + 1(-\frac{1}{2}) = -\frac{1}{2}$

$f(0) = -\frac{1}{2} + \left.\frac{dy}{dx}\right|_{(\frac{1}{2},-\frac{1}{2})} (\Delta x)$

$= -\frac{1}{2} + \frac{3}{2}(-\frac{1}{2})$

$\boxed{f(0) = -\frac{5}{4}}$

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4. Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2 y$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(-1) = 2$.
Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

$\Delta x = \frac{0 - -1}{2} = \frac{1}{2}$

$f(-1) = 2$

$f(-\frac{1}{2}) = 2 + \left.\frac{dy}{dx}\right|_{(-1,2)} (\Delta x)$

$= 2 + [6(-1)^2 - (-1)^2(2)](\frac{1}{2})$

$= 2 + 2$

$= 4$

$f(0) = 4 + \left.\frac{dy}{dx}\right|_{(-\frac{1}{2},4)} (\Delta x)$

$= 4 + [6(-\frac{1}{2})^2 - (-\frac{1}{2})^2(4)]\frac{1}{2}$

$= 4 + \frac{1}{4}$

$\boxed{f(0) = 17/4}$
