Fundamental Theorem of Calculus

If f is cont on $[a,b]$ and F is an antiderivative of f on $[a,b]$,

then $\int_a^b f(x)\,dx = F(b) - F(a)$

ex: $\int_1^2 (x^2-3)\,dx = \left(\frac{1}{3}x^3 - 3x\right)\Big|_1^2$

$= \frac{1}{3}(2)^3 - 3(2) - \left[\frac{1}{3}(1)^3 - 3(1)\right]$ $F(b)-F(a)$

$= \frac{8}{3} - 6 - \left(\frac{1}{3} - 3\right)$

$= \frac{8}{3} - 6 - \frac{1}{3} + 3$

$= \frac{7}{3} - 3$

$= \frac{7}{3} - \frac{9}{3} = \boxed{-\frac{2}{3}}$

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ex: $\int_1^4 \sqrt{x^3}\,dx = \int_1^4 x^{3/2}\,dx$

$$= \frac{1}{5/2} x^{5/2} \Big|_1^4$$

$$= \frac{2}{5} x^{5/2} \Big|_1^4$$

$$= \frac{2}{5}(4)^{5/2} - \frac{2}{5}(1)^{5/2}$$

$$= \frac{2}{5}(\sqrt{4})^5 - \frac{2}{5}$$

$$= \frac{2}{5}(2)^5 - \frac{2}{5}$$

$$= \frac{2}{5} \cdot 32 - \frac{2}{5}$$

$$= \frac{64}{5} - \frac{2}{5} = \boxed{\frac{62}{5}}$$

ex: $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du = \int_{-2}^{-1} (u - u^{-2})\, du$

$= \left(\frac{1}{2}u^2 - \frac{1}{-1}u^{-1}\right)\Big|_{-2}^{-1}$

$= \left(\frac{1}{2}u^2 + u^{-1}\right)\Big|_{-2}^{-1}$

$= \left(\frac{1}{2}u^2 + \frac{1}{u}\right)\Big|_{-2}^{-1}$

$= \frac{1}{2}(-1)^2 + \frac{1}{-1} - \left[\frac{1}{2}(-2)^2 + \frac{1}{-2}\right]$

$= \frac{1}{2} - 1 - \left(2 - \frac{1}{2}\right)$

$= \frac{1}{2} - 1 - 2 + \frac{1}{2}$

$= \frac{1}{2} - 3 + \frac{1}{2}$

$= 1 - 3 = \boxed{-2}$

ex: $\int_0^{\pi/4} \sec^2 x \, dx$

$$= \tan x \Big|_0^{\pi/4}$$

$$= \tan \pi/4 - \tan 0$$

$$= \frac{\sin \pi/4}{\cos \pi/4} - \frac{\sin 0}{\cos 0}$$

$$= \frac{\sqrt{2}/2}{\sqrt{2}/2} - \frac{0}{1}$$

$$= 1 - 0 = \boxed{1}$$

ex: $\int_0^2 (2-x)\sqrt{x}\,dx = \int_0^2 (2\sqrt{x} - x\sqrt{x})\,dx$

$$= \int_0^2 (2x^{1/2} - x^{3/2})\,dx$$

$$= \left[2\left(\frac{1}{3/2}x^{3/2}\right) - \frac{1}{5/2}x^{5/2}\right]\Bigg|_0^2$$

$$= \left(\frac{4}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right)\Bigg|_0^2$$

$$= \frac{4}{3}(2)^{3/2} - \frac{2}{5}(2)^{5/2} - \left(\frac{4}{3}(0)^{3/2} - \frac{2}{5}(0)^{5/2}\right)$$

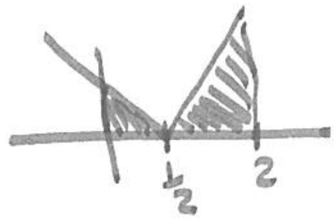
$$= \frac{4}{3}\sqrt{8} - \frac{2}{5}\sqrt{32} - 0$$

$$= \frac{4}{3}\cdot 2\sqrt{2} - \frac{2}{5}\cdot 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2}}{15} - \frac{24\sqrt{2}}{15} = \boxed{\frac{16\sqrt{2}}{15}}$$

ex: $\int_0^2 |2x-1|\,dx$



$$= \int_0^{1/2} -(2x-1)\,dx + \int_{1/2}^2 (2x-1)\,dx$$

$$= \int_0^{1/2} (-2x+1)\,dx + \int_{1/2}^2 (2x-1)\,dx$$

$$= \left[-2\left(\tfrac{1}{2}x^2\right)+x\right]\Big|_0^{1/2} + \left[2\left(\tfrac{1}{2}x^2\right)-x\right]\Big|_{1/2}^2$$

$$= (-x^2+x)\Big|_0^{1/2} + (x^2-x)\Big|_{1/2}^2$$

$$= -\left(\tfrac{1}{2}\right)^2+\tfrac{1}{2}-(-0^2+0) + 2^2-2-\left(\left(\tfrac{1}{2}\right)^2-\tfrac{1}{2}\right)$$

$$= -\tfrac{1}{4}+\tfrac{1}{2} + 4-2-\tfrac{1}{4}+\tfrac{1}{2}$$

$$= \cancel{6+\tfrac{1}{2}} - 2 + 1 - \tfrac{1}{2} = 3 - \tfrac{1}{2}$$

$$= \boxed{\tfrac{5}{2}}$$