Practice Examples

1. $\int x^3\,dx$

$= \frac{1}{4}x^4 + C$

2. $\int (x^5 + 3x^2)\,dx$

$= \int x^5\,dx + 3\int x^2\,dx$

$= \frac{1}{6}x^6 + 3\left(\frac{1}{3}x^3\right) + C$

$= \frac{1}{6}x^6 + x^3 + C$

3. $\int 6\sqrt{x}\,dx$

$= \int 6x^{1/2}\,dx$

$= 6\int x^{1/2}\,dx$

$= 6\left(\frac{1}{3/2}x^{3/2}\right) + C$

$= 6\left(\frac{2}{3}x^{3/2}\right) + C$

$= 4x^{3/2} + C$

or $4\sqrt{x^3} + C$

4. $\int \frac{1}{3x^2}\,dx$

$= \int \frac{1}{3}\cdot\frac{1}{x^2}\,dx$

$= \frac{1}{3}\int \frac{1}{x^2}\,dx$

$= \frac{1}{3}\int x^{-2}\,dx$

$= \frac{1}{3}\cdot\frac{1}{-1}x^{-1} + C$

$= -\frac{1}{3}x^{-1} + C$

or $-\frac{1}{3x} + C$

5. $\int \frac{x^4+3x^2+1}{x}\,dx$

$= \int (x^4 + 3x^2 + 1)x^{-1}\,dx$

$= \int (x^3 + 3x + x^{-1})\,dx$

$= \int \left(x^3 + 3x + \frac{1}{x}\right)dx$

$= \frac{1}{4}x^4 + 3\left(\frac{1}{2}x^2\right) + \ln|x| + C$

$= \frac{1}{4}x^4 + \frac{3}{2}x^2 + \ln|x| + C$

6. $\int (\sin x - \csc x \cot x + x^2)\,dx$

$= \int \sin x\,dx - \int \csc x \cot x\,dx + \int x^2\,dx$

$= -\cos x - (-\csc x) + \frac{1}{3}x^3 + C$

$= -\cos x + \csc x + \frac{1}{3}x^3 + C$

7. $\int \left(4\sqrt[3]{x} - 5\sqrt{x^3} - \frac{1}{\sqrt{x}}\right)dx$

$= \int (4x^{1/3} - 5x^{3/2} - x^{-1/2})\,dx$

$= 4\left(\frac{1}{4/3}x^{4/3}\right) - 5\left(\frac{1}{5/2}x^{5/2}\right) - \frac{1}{1/2}x^{1/2} + C$

$= 4\left(\frac{3}{4}x^{4/3}\right) - 5\left(\frac{2}{5}x^{5/2}\right) - 2x^{1/2} + C$

$= 3x^{4/3} - 2x^{5/2} - 2x^{1/2} + C$

or $3\sqrt[3]{x^4} - 2\sqrt{x^5} - 2\sqrt{x} + C$

8. $\int (8e^x + 2x^3)\,dx$

$= 8e^x + 2\left(\frac{1}{4}x^4\right) + C$

$= 8e^x + \frac{1}{2}x^4 + C$

9. $\int \left(\frac{4}{x} + \frac{3}{x^2} - \frac{1}{x^3}\right) dx$

$= \int (4 \cdot \frac{1}{x} + 3x^{-2} - x^{-3})\, dx$

$= 4\ln|x| + 3\left(\frac{1}{-1}x^{-1}\right) - \frac{1}{-2}x^{-2} + C$

$= 4\ln|x| - 3x^{-1} + \frac{1}{2}x^{-2} + C$

or $4\ln|x| - \frac{3}{x} + \frac{1}{2x^2} + C$

10. $\int (x^2 + 3)(x + 2)\, dx$

$= \int (x^3 + 2x^2 + 3x + 6)\, dx$

$= \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{3}{2}x^2 + 6x + C$