DATE:_____

Antidifferentiation

For each problem, find $f(x)$.

1. $f'(x) = x^2$ $f(x) = \frac{1}{3}x^3 + 2$ (some constant)

$f(x) = \frac{1}{3}x^3 + 2$, $f'(x) = \frac{1}{3} \cdot 3x^2 = x^2$ ☺

2. $f'(x) = -\sin x$ $f(x) = \cos x + 7$ (some constant)

$\frac{d}{dx}(\cos x) = -\sin x$

3. $f'(x) = x^3 + 3x^2 + 5$ $f(x) = \frac{1}{4}x^4 + x^3 + 5x - 42$ (some constant)

$F'(x) = \frac{1}{4} \cdot 4x^3 + 3x^2 + 5 = x^3 + 3x^2 + 5$

Rules for Integrating (Antideriving)

① $\int 0\,dx = C$ (constant)

② $\int k\,dx = kx + C$ (# → constant)

Example: $\int 3\,dx = 3x + C$

Example: $\int \pi\,d = \pi x + C$

③ $\int kf(x)\,dx = k\int f(x)\,dx$ (# → constant)

④ $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

⑤ Power Formulas

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

Example: $\int (x^5 + x^4 + 3x)\,dx$

$= \int x^5\,dx + \int x^4\,dx + \int 3x\,dx$

$= \int x^5\,dx + \int x^4\,dx + 3\int x\,dx$

$= \frac{1}{6}x^6 + \frac{1}{5}x^5 + 3\left(\frac{1}{2}x^2\right) + C$

$= \frac{1}{6}x^6 + \frac{1}{5}x^5 + \frac{3}{2}x^2 + C$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

← need abs value, can't ln neg #s.

think backwards: $f(x) = ?$, $f'(x) = \frac{1}{x}$ ☺

Example: $\int \left(\frac{1}{x} + \frac{1}{x^2}\right)\,dx$

$= \int \frac{1}{x}\,dx + \int \frac{1}{x^2}\,dx$

$= \int \frac{1}{x}\,dx + \int x^{-2}\,dx$

$= \ln|x| + \frac{1}{-1}x^{-1} + C$

$= \ln|x| - x^{-1} + C$ or $\ln|x| - \frac{1}{x} + C$

⑥ Exponential Formulas

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{1}{\ln a} \cdot a^x + C$

Example: $\int (3e^x + 3^x)\,dx$

$= 3\int e^x dx + \int 3^x dx$

$= 3e^x + \frac{1}{\ln 3} \cdot 3^x + C$

or $3e^x + \frac{3^x}{\ln 3} + C$

⑦ Trig Formulas

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,d = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

Example: $\int (\cos x - \sin x)\,dx$

$= \int \cos x\,dx - \int \sin x\,dx$

$= \sin x - (-\cos x) + C$

$= \sin x + \cos x + C$