

6.2 Dot Product of Vectors

Target 8C: Calculate and use properties of the Dot Product

Review of Prior Concepts

1. Let $\mathbf{u} = \langle 2, -1 \rangle$. Sketch \mathbf{u} and $2\mathbf{u}$.

2. Find \mathbf{AB} , if $\mathbf{A} = \begin{bmatrix} 1 & -3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$



$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 1(4) + -3(2) \end{bmatrix} \\ &= \begin{bmatrix} -2 \end{bmatrix} \end{aligned}$$

$(1 \times 2)(2 \times 1)$
 $= 1 \times 1$
one by one

Dot Product

If $\vec{u} = \langle a, b \rangle$ and $\vec{v} = \langle c, d \rangle$, then

$$\vec{u} \cdot \vec{v} = ac + bd$$

constant #
"scalar"

$\vec{u} = [a \ b], \vec{v} = \begin{bmatrix} c \\ d \end{bmatrix}$, then
 $uv = [ac + bd]$

Example

Given $\vec{u} = \langle 1, 3 \rangle$ and $\vec{v} = \langle 4, 5 \rangle$, find $\vec{u} \cdot \vec{v}$.

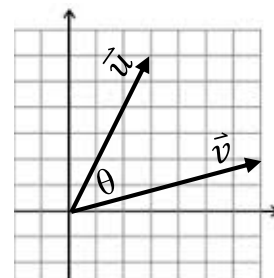
$$\begin{aligned} \vec{u} \cdot \vec{v} &= 1(4) + 3(5) \\ &= \boxed{19} \end{aligned}$$

Angle Between Two Vectors

See how this formula comes about at:

<https://youtu.be/eLMLJkcllBs>

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\mathbf{u}| |\mathbf{v}|}$$



Example

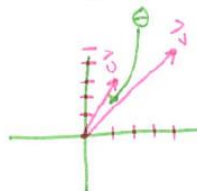
Given $\vec{u} = \langle 1, 3 \rangle$ and $\vec{v} = \langle 4, 5 \rangle$, find the angle between \vec{u} and \vec{v} .

$$\cos \theta = \frac{1(4) + 3(5)}{(\sqrt{10})(\sqrt{41})} \rightarrow \frac{\vec{u} \cdot \vec{v}}{|\mathbf{u}| |\mathbf{v}|}$$

$$\begin{aligned} \cos \theta &= \frac{19}{(\sqrt{10})(\sqrt{41})} \\ \theta &= \cos^{-1} \left(\frac{19}{\sqrt{10}\sqrt{41}} \right) \end{aligned}$$

$$\theta = \boxed{20.225^\circ}$$

$$\begin{aligned} |\vec{u}| &= \sqrt{1^2 + 3^2} \\ &= \sqrt{10} \end{aligned} \quad \begin{aligned} |\vec{v}| &= \sqrt{4^2 + 5^2} \\ &= \sqrt{41} \end{aligned}$$



Now you try,

1. Given $\vec{u} = \langle 2, -4 \rangle$, and $\vec{v} = \langle -8, 7 \rangle$, find $\vec{u} \cdot \vec{v}$.

$$\begin{aligned}\vec{u} \cdot \vec{v} &= 2(-8) + (-4)(7) \\ &= -16 + -28 \\ &= \boxed{-44}\end{aligned}$$

2. Given $\vec{u} = 4i - 11j$ and $\vec{v} = -3j$, find $\vec{u} \cdot \vec{v}$.

$$\begin{aligned}\vec{u} \cdot \vec{v} &= 4(0) + -11(-3) \\ &= \boxed{33}\end{aligned}$$

$\vec{u} = \langle 4, -11 \rangle$ $\vec{v} = \langle 0, -3 \rangle$

3. Given $\vec{u} = \langle -3, 8 \rangle$ and $\vec{v} = \langle -1, -9 \rangle$, find the angle between \vec{u} and \vec{v} .

$$\begin{aligned}\cos \theta &= \frac{-3(-1) + 8(-9)}{\sqrt{73} \cdot \sqrt{82}} \\ \theta &= \cos^{-1} \left(\frac{-69}{\sqrt{73} \cdot \sqrt{82}} \right)\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{(-3)^2 + 8^2} \\ &= \sqrt{73}\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{(-1)^2 + (-9)^2} \\ &= \sqrt{82}\end{aligned}$$

$$\theta = \boxed{153.104^\circ}$$

4. Given $\vec{u} = \langle -2, 0 \rangle$ and $\vec{v} = \langle 0, 5 \rangle$, find the angle between \vec{u} and \vec{v} .

$$\begin{aligned}\cos \theta &= \frac{-2(0) + 0(5)}{2 \cdot 5} \\ \cos \theta &= \frac{0}{10} \\ \cos \theta &= 0 \\ \theta &= 90^\circ\end{aligned}$$

$$\begin{aligned}|\vec{u}| &= \sqrt{0^2 + 2^2} \\ &= 2\end{aligned}$$

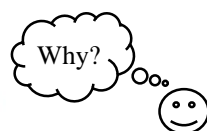
$$\begin{aligned}|\vec{v}| &= \sqrt{0^2 + 5^2} \\ &= 5\end{aligned}$$

Orthogonal Vectors

If $\vec{u} \cdot \vec{v} = 0$, then the vectors are orthogonal.

↳ "perpendicular"

"the 2 vectors make a 90° angle"



Example:

(Non-calculator)

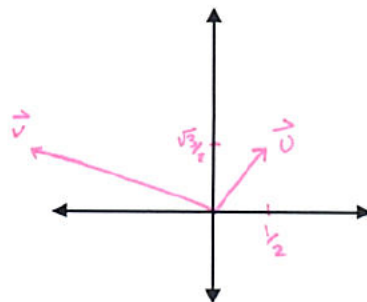
Given $\vec{u} = \langle \cos \frac{\pi}{3}, \sin \frac{\pi}{3} \rangle$ and $\vec{v} = \langle 3 \cos \frac{5\pi}{6}, 3 \sin \frac{5\pi}{6} \rangle$, find the angle between \vec{u} and \vec{v} .

$$\begin{aligned}\vec{u} &= \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle & \vec{v} &= \left\langle 3 \left(\frac{\sqrt{3}}{2} \right), 3 \left(\frac{1}{2} \right) \right\rangle \\ & & \vec{v} &= \left\langle -\frac{3\sqrt{3}}{2}, \frac{3}{2} \right\rangle\end{aligned}$$

$$\begin{aligned}\vec{u} \cdot \vec{v} &= \frac{1}{2} \left(-\frac{3\sqrt{3}}{2} \right) + \frac{\sqrt{3}}{2} \left(\frac{3}{2} \right) \\ &= -\frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}\end{aligned}$$

$$= 0$$

$$\therefore, \vec{u} \perp \vec{v} \text{ are orthogonal, so } \theta = \boxed{90^\circ}$$



More Practice

Dot Product

<https://www.mathsisfun.com/algebra/vectors-dot-product.html>

<https://betterexplained.com/articles/vector-calculus-understanding-the-dot-product/>

<https://youtu.be/KDHuWxy53uM>

<https://youtu.be/98C7iv8OcnI>

Angle Between Vectors

<http://onlinemath.com/math/library/vector/angl/>

<https://youtu.be/WDdR5s0C4cY>

<https://youtu.be/4WxniMJYySc>

Homework Assignment

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