DATE: _____

# AP Multiple-Choice & FRQ Questions

**1.** $\int(\sec^2 x - 2)dx =$ $\int \sec^2 x\,dx - \int 2\,dx$

$= \tan x - 2x + C$

**(A)** $\tan x + C$

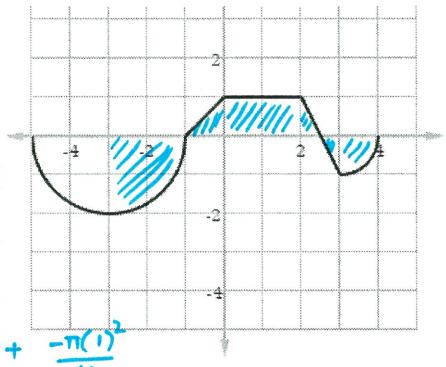
**(B)** $\tan x - 2x + C$

**(C)** $\frac{\tan^3 x}{3} - x + C$

**(D)** $2\tan x \sec^2 x - x + C$

---

**2.** The graph of $f$ consists of line segments, a semicircle, and a quarter circle as shown in the graph to the right. Given $g(x) = \int_{-3}^{x} f(t)\,dt$, what is the value of $g(4)$?




**(A)** $\frac{10-5\pi}{4}$

**(B)** $\frac{10+5\pi}{4}$

**(C)** $\frac{12+5\pi}{4}$

**(D)** $\frac{12-5\pi}{4}$

$g(4) = \int_{-3}^{4} f(t)\,dt$

$= -\frac{\pi(2)^2}{4} + \frac{1}{2}(1)(1) + 2(1) + \frac{1}{2}(\frac{1}{2})(1) + \frac{1}{2}(\frac{1}{2})(-1) + \frac{-\pi(1)^2}{4}$

$= -\pi + \frac{1}{2} + 2 - \frac{\pi}{4} = \frac{-4\pi + 2 + 8 - \pi}{4} = \frac{10-5\pi}{4}$

---

**3.** If $\int_1^3 f(x)dx = k$ and $\int_1^7 f(x)\,dx = -4$, what is the value of $\int_7^3 (x + f(x))\,dx$?

**(A)** $k + 4$

**(B)** $k - 4$

**(C)** $16 - k$

**(D)** $-16 + k$

$\int_7^3 (x + f(x))\,dx = -\int_3^7 (x + f(x))\,dx$

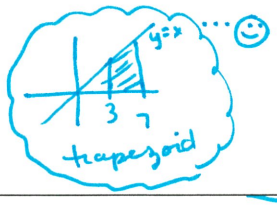
$= -\int_3^7 x\,dx - \int_3^7 f(x)\,dx$

$= -\frac{1}{2}(7+3)(4) - \left[\int_1^7 f(x)dx - \int_1^3 f(x)dx\right]$

$= -20 - (-4 - k)$

$= -20 + 4 + k$

$= -16 + k$




---

**4.** What is $\int \frac{x-3}{x}dx$ ?

**(A)** $1 - 3\ln x + C$

**(B)** $x - 3\ln|x| + C$

**(C)** $1 + \frac{3}{x^2} + C$

**(D)** $\frac{x^2-3x}{x^2} + C$

$\int \frac{x-3}{x}dx = \int (x-3)x^{-1}\,dx$

$= \int (1 - 3x^{-1})\,dx$

$= \int 1\,dx - 3\int \frac{1}{x}dx$

$= x - 3\ln|x| + C$

OR

$\int \frac{x-3}{x}dx$

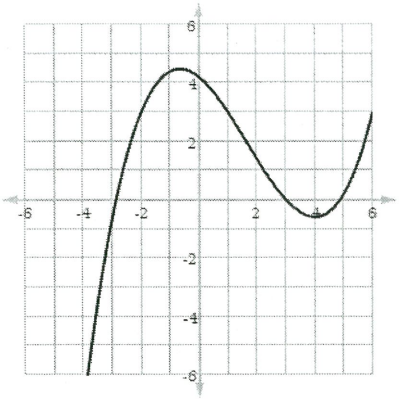
$= \int (\frac{x}{x} - \frac{3}{x})dx$

$= \int (1 - \frac{3}{x})dx$

$= \int 1\,dx - 3\int \frac{1}{x}dx$

$= x - 3\ln|x| + C$

**5.** The graph of $f(x)$ is pictured to the right. Which of the following statements about $\int_0^4 f(x)\,dx$ are true?




**I.** A left endpoint approximation is greater than $\int_0^4 f(x)\,dx$.

**II.** A right endpoint approximation is less than $\int_0^4 f(x)\,dx$.

**(A)** I only **(B)** II only **(C)** I and II **(D)** None are true

I. true b/c $f(x)$ inc. on $(0,4)$

II. true b/c $f(x)$ inc. on $(0,4)$

---

**6.** $\int \left(2e^x + \frac{5}{x^2}\right) dx =$

$2\int e^x\,dx + 5\int x^{-2}\,dx$

$= 2e^x - 5x^{-1} + C$

$= 2e^x - \frac{5}{x} + C$

**(A)** $e^{2x} - \frac{5}{x} + C$

**(B)** $e^{2x} - \frac{10}{x^3} + C$

**(C)** $2e^x - \frac{5}{x} + C$

**(D)** $2e^x + \frac{10}{x^3} + C$

---

**7.** A particle moves along the $x$-axis with a velocity given by $v(t) = 2 + \sin t$. When $t = 0$, the particle is at $x = -2$. Where is the particle when $t = \pi$?

$v(t) = x'(t)$

so, $x(t) = \int v(t)$

**(A)** $\pi$

**(B)** $2\pi$

**(C)** $\pi - 1$

**(D)** $\pi - 2$

$x(t) = \int v(t)\,dt$

$= \int (2 + \sin t)\,dt$

$x(t) = 2t - \cos t + C$

$x(0) = 2(0) - \cos 0 + C$

$-2 = -1 + C$

$-1 = C$

$\rightarrow x(t) = 2t - \cos t - 1$

$x(\pi) = 2\pi - \cos\pi - 1$

$= 2\pi - (-1) - 1$

$= 2\pi$

---

**8.** The volume of water in a water tank changes at a rate given by the equation: $V'(t) = \frac{1}{3}t^2 - \sqrt{t} + 5$, where $V'(t)$ is measure in gallons per minute and $t$ is measured in minutes. If the water tank initially has 45 gallons of water, how much water is in the tank after 9 hours?

$V(0) = 45$

$V(t) = ?$

$V(9) = ?$

$V(t) = \int V'(t)\,dt$

$= \int \left(\frac{1}{3}t^2 - \sqrt{t} + 5\right) dt$

$= \frac{1}{3}\int t^2\,dt - \int t^{1/2}\,dt + \int 5\,dt$

$= \frac{1}{3}\left(\frac{1}{3}t^3\right) - \frac{1}{3/2}t^{3/2} + 5t + C$

$V(t) = \frac{1}{9}t^3 - \frac{2}{3}t^{3/2} + 5t + C$

$V(0) = \frac{1}{9}(0)^3 - \frac{2}{3}(0)^{3/2} + 5(0) + C$

$45 = C$

$V(t) = \frac{1}{9}t^3 - \frac{2}{3}t^{3/2} + 5t + 45$

$V(9) = \frac{1}{9}(9)^3 - \frac{2}{3}(9)^{3/2} + 5(9) + 45$

$= 81 - \frac{2}{3}(\sqrt{9})^3 + 45 + 45$

$= 81 - 18 + 90 = \boxed{153 \text{ gallons}}$