

## 6.2 Dot Product of Vectors (continued)

Target 8D: Apply properties of vectors to real life situations

## Review of Prior Concepts

1. Given  $\vec{u} = \langle 5, 2 \rangle$  and  $\vec{v} = \langle -4, 3 \rangle$ , find the angle between the two vectors.

$$\cos \theta = \frac{5(-4) + 2(3)}{(\sqrt{29})(5)}$$

$$\cos \theta = \frac{-14}{5\sqrt{29}}$$

$$\theta = \cos^{-1}\left(\frac{-14}{5\sqrt{29}}\right)$$

$$\theta = 121.329^\circ$$

$$|\vec{u}| = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$|\vec{v}| = \sqrt{(-4)^2 + 3^2} = 5$$

2. Find the value of  $x$  that would make  $\vec{u} = \langle 5, 2 \rangle$  and  $\vec{v} = \langle x, 3 \rangle$  orthogonal.

$$\vec{u} \cdot \vec{v} = 5(x) + 2(3)$$

$$0 = 5x + 6$$

$$-6 = 5x$$

$$-\frac{6}{5} = x \quad \text{or} \quad -1.2$$

$$\vec{u} \cdot \vec{v} = 0$$

## Work

$$\text{Work} = \text{Force} \cdot \text{Distance}$$

$$W = F \cdot d$$

$$d = s$$

## Examples:

1. Abigail lifts a book that weighs 2 lbs from the floor onto a shelf that is 4 feet high. How much work did she do?

$$W = F \cdot d$$

$$= (2 \text{ lbs})(4 \text{ ft})$$

$$W = 8 \text{ ft} \cdot \text{lbs}$$

in science...  
F in Newtons  
d in meters  
 $\therefore W = \text{Newton} \cdot \text{meters} = \text{Joules}$

2. Juan is sitting on a desk. The combined weight of Juan and the desk is 155 pounds. How much work must Oswald do to lift Juan and the desk 6 ft high?

$$W = F \cdot d$$

$$= 155(6)$$

$$= 930 \text{ ft} \cdot \text{lbs}$$

3. How much work must Karen do to lift a 100 pound sack of potatoes 3 feet?

$$W = F \cdot d$$

$$= 100 \cdot 3$$

$$= 300 \text{ ft} \cdot \text{lbs}$$

Work & Force with Angular Direction Examples

1. Jose is sitting on a sled on the side of a hill that is inclined at a  $35^\circ$  angle. Jose and the sled weigh 140 lbs. Alejandro needs to use what force to pull Jose up the hill?

$\sin 35^\circ = \frac{|\vec{F}|}{140}$   
 $140 \sin 35^\circ = |\vec{F}|$   
 $80.301 \text{ lbs} = |\vec{F}|$

2. Mandy is pulling a box up a hill that weighs 20 lbs. The hill is at a  $75^\circ$  angle. What force does she need to use?

$\sin 75^\circ = \frac{|\vec{F}|}{20}$   
 $20 \sin 75^\circ = |\vec{F}|$   
 $19.319 \text{ lbs} = |\vec{F}|$

3. Oscar is dragging his luggage through the airport at an angle of  $65^\circ$  with a force of 400N over a distance of 47m. How much work did he do?

$\vec{F} = \langle |\vec{F}| \cos \theta, |\vec{F}| \sin \theta \rangle$   
 $|\vec{F}_H|$  horizontal force       $|\vec{F}_V|$  vertical force  
 $W = \vec{F}_H \cdot d$   
 $= (400 \cos 65^\circ)(47 \text{ m})$   
 $= 7945.223 \text{ N}\cdot\text{m}$   
 $7945.223 \text{ Joule}$

4. Find the work done by a 10 pound force acting in the direction  $\langle 1,2 \rangle$  in moving an object 3 feet from  $(0,0)$  to  $(3,0)$ .

$\cos \theta = \frac{u \cdot v}{|u||v|}$   
 $\cos \theta = \frac{1(3) + 2(0)}{\sqrt{5} \cdot \sqrt{9}}$   
 $\cos \theta = \frac{3}{\sqrt{5} \cdot 3}$   
 $\cos \theta = \frac{1}{\sqrt{5}}$   
 OR  $\cos \theta = \frac{1}{\sqrt{5}}$   
 $|\vec{F}| = 10 \text{ lb}$   
 $|\vec{F}_H| = |\vec{F}| \cos \theta = 10 \left(\frac{1}{\sqrt{5}}\right)$   
 $|\vec{F}_H| = \frac{10}{\sqrt{5}}$   
 $W = |\vec{F}_H| \cdot d$   
 $= \frac{10}{\sqrt{5}} \cdot 3$   
 $= 13.416 \text{ ft}\cdot\text{lbs}$

**More Practice****Work & Force**

[https://www.varsitytutors.com/hotmath/hotmath\\_help/topics/solving-problems-with-vectors](https://www.varsitytutors.com/hotmath/hotmath_help/topics/solving-problems-with-vectors)

<https://www.khanacademy.org/math/precalculus/vectors-precac/applications-of-vectors/v/vector-component-in-direction>

<http://www.physicsclassroom.com/class/energy/Lesson-1/Calculating-the-Amount-of-Work-Done-by-Forces>

<https://www.mansfieldct.org/Schools/MMS/staff/hand/work=fxd.htm>

<http://www.uwgb.edu/fenclh/problems/energy/1/>

<https://youtu.be/WSY4HzWZllo>

<https://youtu.be/tZOBPEwshb8>

<https://youtu.be/EKyWQKi76uo>

**Homework Assignment**

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