DATE: ____________

6.2 Anti-differentiation: Vectors/Parametric

Net distance (displacement) over $t = a$ to $t = b$ → distance btn object @ t=a to t=b

difference btn position @ t=a and position @ t=b

$$\underbrace{x(b) - x(a)}_{\text{net distance}} = \int_a^b x'(t)\,dt$$

(where $x'(t) = v(t)$)... ☺

Position @ $t = a$ = initial position + net distance

$$x(a) = x(t_0) + \int_{t_0}^{a} x'(t)\,dt$$

or

$$y(a) = y(t_0) + \int_{t_0}^{a} y'(t)\,dt$$

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS

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1. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = \sqrt{3t}$ and $\frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right)$. The particle is at position $(1,5)$ at time $t = 4$. Find position vector of the particle at time $t = 0$.

$(x(4), y(4))$ — initial time

$$x(0) = x(4) + \int_4^0 \frac{dx}{dt}\,dt$$

$$= 1 + \int_4^0 \sqrt{3t}\,dt$$

$$= -8.238$$

$$y(0) = y(4) + \int_4^0 \frac{dy}{dt}\,dt$$

$$= 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right)dt$$

$$= 1.601$$

$\boxed{\text{Position @ } t=0 \text{ is } \langle -8.238, 1.601\rangle}$

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2. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1,8)$. At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

→ $y - y_1 = m(x - x_1)$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{dy/dt\,|_{t=2}}{dx/dt\,|_{t=2}} = \frac{-7}{3+\cos(2)^2} = -2.983$$

$x(2) = 1$

$y(2) = 8$

$\boxed{y - 8 = -2.983(x - 1)}$

3. A particle moves in the xy-plane in such as way that its velocity vector is $\langle 1+t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5,0 \rangle$, find the position of the particle at $t = 2$.

initial time → $(x(0), y(0))$

$x(2) = x(0) + \int_0^2 x'(t)\,dt$

$= 5 + \int_0^2 (1+t)\,dt$

$= 5 + \left(\frac{1}{2}t^2 + t\right)\Big|_0^2$

$= 5 + \left(\frac{1}{2}(2)^2 + 2\right) - \left(\frac{1}{2}(0)^2 + 0\right)$

$= 5 + (2+2) - 0$

$= 9$

$y(2) = 0 + \int_0^2 t^3\,dt$

$= \frac{1}{4}t^4 \Big|_0^2$

$= \frac{1}{4}\left((2)^4 - 0^4\right)$

$= \frac{1}{4} \cdot 16$

$= 4$

$\boxed{\text{Position of particle @ } t=2 \text{ is } \langle 9, 4 \rangle}$

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4. Point $P(x, y)$ moves in the xy-plane in such as way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$. Find the coordinates of P in terms of t given that, when $t = 1$, $x = ln2$ and $y = 0$.

end time   initial time $x(1) = \ln 2$ $y(1) = 0$

$x(t) = x(1) + \int_1^t \frac{1}{w+1}\,dw$

$u = w+1$, $du = dw$, $u(t) = \frac{1}{t+1}$, $u(1) = \frac{1}{2}$

$= \ln 2 + \int_{\frac{1}{2}}^{\frac{1}{t+1}} \frac{1}{u}\,du$

$= \ln 2 + \ln|u| \Big|_{\frac{1}{2}}^{\frac{1}{t+1}}$

$= \ln 2 + \ln\left(\frac{1}{t+1}\right) - \ln\frac{1}{2}$

$= \ln 2 + \ln 1 - \ln(t+1) - (\ln 1 - \ln 2)$

$= \ln 2 + \ln 1 + \ln(t+1) - \ln 1 + \ln 2$

$= 2\ln 2 + \ln(t+1)$

or $= \ln 2^2 + \ln(t+1)$

$= \ln(4(t+1))$

$y(t) = y(1) + \int_1^t 2w\,dw$

$= 0 + w^2 \Big|_1^t$

$= t^2 + 1$

$\boxed{\text{coordinates of } P \text{ are } \left(\ln(4(t+1)),\ t^2+1\right)}$