

ANTIDERIVATIVES AND THE FUNDAMENTAL THEOREMS OF CALCULUS

20. In order to evaluate $\int t \sin(t^2) dt$ using substitution, you would probably use $u =$

- A. t^2
- B. $\cos(t^2)2t dt$
- C. $2t dt$
- D. $\sin(t^2)$
- E. t

22. In order to evaluate $\int \frac{x}{\sqrt{1-x^2}} dx$ using substitution, you would set $u =$

- A. x
- B. $1-x^2$
- C. $1-x$
- D. $\frac{1}{\sqrt{1-x^2}}$
- E. Substitution won't work.

↳ rewrite to see the chain better
 $\int x(1-x^2)^{-1/2} dx$

23. If $g(x) = x\sqrt{9-x^2}$, the antiderivative $G(x) = \int x(9-x^2)^{1/2} dx$ $u = 9-x^2$

- A. $\frac{3}{2}x^2 - \frac{1}{3}x^3 + C$ $= \int x u^{1/2} \cdot \frac{du}{-2x}$ $\frac{du}{dx} = -2x$
- B. $\frac{3}{2}x^2 - x^3 + C$ $= \int -\frac{1}{2} u^{1/2} du$ $\frac{du}{-2x} = dx$
- C. $-\frac{1}{2}((9-x^2)^{3/2}) + C$ $= -\frac{1}{2}(\frac{2}{3}u^{3/2}) + C$
- D. $\frac{(9-x^2)^{3/2}}{3} + C$ $= -\frac{1}{3}u^{3/2} + C$
- E. $-\frac{1}{3}(\sqrt{9-x^2})^3 + C$ $= -\frac{1}{3}(9-x^2)^{3/2} + C$

24. Given that $g'(t) = (\sin t)(5 + 5 \cos t)^3$, find $g(t)$ if $g(0) = 0$.

- A. $g(t) = -\frac{1}{4}(5 + 5 \cos t)^4$
- B. $g(t) = -\frac{1}{4}(5 + 5 \cos t)^4 + 2,500$
- C. $g(t) = \frac{1}{4}(5 + 5 \cos t)^4 - 2,500$
- D. $g(t) = -\frac{1}{20}(5 + 5 \cos t)^4 + 500$
- E. $g(t) = \frac{1}{20}(5 + 5 \cos t)^4 - 500$

$g(t) = \int (\sin t)(5 + 5 \cos t)^3 dt$
 $= \int \sin t (u)^3 \cdot \frac{du}{-5 \sin t}$ $u = 5 + 5 \cos t$
 $= -\frac{1}{5} \int u^3 du$ $\frac{du}{dt} = -5 \sin t$
 $= -\frac{1}{5} \cdot \frac{1}{4} u^4 + C$ $\frac{du}{-5 \sin t} = dt$
 $g(t) = -\frac{1}{20}(5 + 5 \cos t)^4 + C$
 $g(0) = -\frac{1}{20}(5 + 5 \cos 0)^4 + C$
 $0 = -\frac{1}{20}(10^4) + C$

$$25. \int 3x(x^2 + 7)^3 dx = \int 3x \cdot u^3 \cdot \frac{du}{2x} \quad u = x^2 + 7$$

$$\boxed{\text{A.}} \quad \frac{3}{8}(x^2 + 7)^4 + C = \frac{3}{2} \int u^3 du \quad \frac{du}{dx} = 2x$$

$$\text{B.} \quad \frac{3}{4}(x^2 + 7)^4 + C = \frac{3}{2} \left(\frac{1}{4} u^4 \right) + C \quad \frac{du}{2x} = dx$$

$$\text{C.} \quad \frac{1}{4}(x^2 + 7)^4 + C = \frac{3}{8} (x^2 + 7)^4 + C$$

$$\text{D.} \quad 3(x^2 + 7)^4 + C$$

E. None of these.

$$26. \int \frac{x^2}{(2x^3 - 12)^4} dx = \int x^2 (2x^3 - 12)^{-4} dx \quad u = 2x^3 - 12$$

$$\text{A.} \quad -(2x^3 - 12)^{-3} + C = \int x^2 u^{-4} \cdot \frac{du}{6x^2} \quad \frac{du}{dx} = 6x^2$$

$$\boxed{\text{B.}} \quad \frac{-1}{18(2x^3 - 12)^3} + C = \frac{1}{6} \int u^{-4} du \quad \frac{du}{6x^2} = dx$$

$$\text{C.} \quad \frac{-1}{3(2x^3 - 12)^3} + C = \frac{1}{6} \left(-\frac{1}{3} u^{-3} \right) + C$$

$$\text{D.} \quad -\frac{1}{4}(2x^3 - 12)^{-3} + C = -\frac{1}{18} (2x^3 - 12)^{-3} + C$$

E. None of these.

$$30. \int \sec^2 x \tan x dx = \int \sec^2 x \cdot u \cdot \frac{du}{\sec^2 x} \quad u = \tan x$$

$$\text{A.} \quad \frac{1}{3} \sec^3 x + C = \int u du \quad \frac{du}{dx} = \sec^2 x$$

$$\text{B.} \quad \tan^2 x + C = \frac{1}{2} u^2 + C \quad \frac{du}{\sec^2 x} = dx$$

$$\boxed{\text{C.}} \quad \frac{1}{2} \tan^2 x + C = \frac{1}{2} \tan^2 x + C$$

$$\text{D.} \quad 2 \sec^2 x + C$$

E. None of these.

$$34. \int \sin(4x) \sec^6(4x) dx = \int \sin(4x) [\cos(4x)]^{-6} dx \quad u = \cos 4x$$

$$\text{A.} \quad \frac{1}{7} \sec^7(4x) + C = \int \sin 4x \cdot u^{-6} \cdot \frac{du}{-4 \sin 4x} \quad \frac{du}{dx} = -\sin(4x) \cdot 4$$

$$\boxed{\text{B.}} \quad \frac{1}{20} \sec^5(4x) + C = -\frac{1}{4} \int u^{-6} du \quad \frac{du}{-4 \sin 4x} = dx$$

$$\text{C.} \quad \frac{1}{28} \sec^7(4x) + C = -\frac{1}{4} \left(\frac{1}{-5} u^{-5} \right) + C$$

$$\text{D.} \quad \frac{1}{28} \sec^5(4x) \tan(4x) + C = \frac{1}{20} \cos^{-5}(4x) + C = \frac{1}{20} \sec^5(4x) + C$$

E. None of these.