Definite Integrals w/ Substitution

x-values

ex: $\int_0^2 x(x^2+1)^3\,dx$

x values

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$u(0) = 0^2 + 1 = 1$

$u(2) = 2^2 + 1 = 5$

u-values

$= \int_1^5 x \cdot u^3 \cdot \frac{du}{2x}$

u-values

$= \frac{1}{2}\int_1^5 u^3\,du$

$= \frac{1}{2}\left(\frac{1}{4}u^4\right)\Big|_1^5$

stay in u's... don't go back to x's

$= \frac{1}{8}(5^4 - 1^4)$

$= \frac{1}{8}(625 - 1)$

$= \frac{624}{8} = \boxed{78}$

* once "U" Leave your ex,
definitely

"U" definitely don't go back to your ex.

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:)

ex: $\int_0^1 x\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

$= \int_1^0 x \cdot \sqrt{u} \cdot \frac{du}{-2x}$

$= -\frac{1}{2}\int_1^0 u^{1/2}\,du$

$u(1) = 1-1^2 = 0$

$u(0) = 1-0^2 = 1$

$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^0$

$= -\frac{1}{3}\left(0^{3/2} - 1^{3/2}\right)$

$= -\frac{1}{3}(-1) = \boxed{\frac{1}{3}}$

ex: $\int_{-\pi/4}^{0} \tan x \sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$\frac{du}{\sec^2 x} = dx$

$= \int_{-1}^{0} u \cdot \sec^2 x \cdot \frac{du}{\sec^2 x}$

$= \int_{-1}^{0} u\,du$

$u(0) = \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$u(-\pi/4) = \tan(-\pi/4) = \frac{\sin(-\pi/4)}{\cos(-\pi/4)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

$= \frac{1}{2}u^2\Big|_{-1}^{0}$

$= \frac{1}{2}\left(0^2 - (-1)^2\right) = \boxed{-\frac{1}{2}}$