DATE: _____

2000 AP Calculus AB FRQ (Non-Calculator)

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

$\frac{dl}{dt} = \sqrt{t+1}$

$\int dl = \int_0^3 \sqrt{t+1}\, dt \qquad u = t+1,\ du = dt,\ u(0) = 1,\ u(3) = 4$

$l = \int_1^4 u^{1/2}\, du$

$= \frac{2}{3} u^{3/2} \Big|_1^4 = \frac{2}{3}(4^{3/2} - 1^{3/2}) = \frac{2}{3}(8-1) = \frac{14}{3}$ gallons

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

initial gallons − leaked gallons + gallons pumped in

$30 - \frac{14}{3} + 8(3)$

$30 + 24 - \frac{14}{3}$

$54 - \frac{14}{3}$

$\frac{162}{3} - \frac{14}{3} = \frac{148}{3}$ gallons

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

$A(t) = 30 + \underbrace{\int_0^t 8\, dx}_{\text{pumped in}} - \underbrace{\int_0^t \sqrt{x+1}\, dx}_{\text{leaked out}}$

$= 30 + \int_0^t (8 - \sqrt{x+1})\, dx$

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

$A'(t)$

$A'(t) = \underbrace{8}_{\text{rate in}} - \underbrace{\sqrt{t+1}}_{\text{rate out}}$

$0 = 8 - \sqrt{t+1}$

$\sqrt{t+1} = 8$

$t + 1 = 64$

$t = 63$ minutes

$A''(t) = -\frac{1}{2}(t+1)^{-1/2}$

$A''(63) = -\frac{1}{2}(63+1)^{-1/2} < 0$

$\therefore$, max @ $t = 63$