① $\int (3x-2)^4\,dx$

$u = 3x-2$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$= \int u^4 \cdot \frac{du}{3}$

$= \frac{1}{3}\int u^4\,du$

$= \frac{1}{3}\cdot\frac{1}{5}u^5 + C$

$= \frac{1}{15}(3x-2)^5 + C$

② $\int \frac{2}{(2x-1)^2+4}\,dx$

$= \int \frac{2}{4\left(\frac{(2x-1)^2}{4}+1\right)}\,dx$

$= \frac{1}{2}\int \frac{1}{\left(\frac{2x-1}{2}\right)^2+1}\,dx$

$= \frac{1}{2}\int \frac{1}{(x-\frac{1}{2})^2+1}\,dx$

$u = x-\frac{1}{2}$

$\frac{du}{dx} = 1$

$du = dx$

$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$

$= \frac{1}{2}\tan^{-1}(u) + C$

$= \frac{1}{2}\tan^{-1}\left(x-\frac{1}{2}\right) + C$

③ $\int 3xe^{x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$= \int 3x \cdot e^u \cdot \frac{du}{2x}$

$= \frac{3}{2}\int e^u\,du$

$= \frac{3}{2}e^u + C$

$= \frac{3}{2}e^{x^2} + C$

④ $\int \frac{5}{(x-4)^5}\,dx$

$= \int 5(x-4)^5\,dx$

$u = x-4$

$du = dx$

$= 5\int u^5\,du$

$= 5\left(\frac{1}{6}u^6\right) + C$

$= \frac{5}{6}u^6 + C$

$= \frac{5}{6}(x-4)^6 + C$

(5) $\int \left(x - \frac{3}{(2x+3)^2}\right) dx$

$= \int x\,dx - \int 3(2x+3)^{-2}\,dx$

$u = 2x+3$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$= \frac{1}{2}x^2 - 3\int u^{-2} \cdot \frac{du}{2}$

$= \frac{1}{2}x^2 - \frac{3}{2}\int u^{-2}\,du$

$= \frac{1}{2}x^2 - \frac{3}{2}(-u^{-1}) + C$

$= \frac{1}{2}x^2 + \frac{3}{2}u^{-1} + C$

$= \frac{1}{2}x^2 + \frac{3}{2(2x+3)} + C$

(6) $\int \frac{e^{2x}}{3+\frac{1}{2}e^{2x}}\,dx$

$u = 3 + \frac{1}{2}e^{2x}$

$\frac{du}{dx} = \frac{1}{2}e^{2x} \cdot 2$

$\frac{du}{e^{2x}} = dx$

$= \int \frac{e^{2x}}{u} \cdot \frac{du}{e^{2x}}$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln\left|3 + \frac{1}{2}e^{2x}\right| + C$

(7) $\int \sec 4x\,dx$

$= \int \sec 4x \cdot \frac{\sec 4x + \tan 4x}{\sec 4x + \tan 4x}\,dx$

$= \int \frac{\sec^2 4x + \sec 4x \tan 4x}{\sec 4x + \tan 4x}\,dx$

$u = \sec 4x + \tan 4x$

$\frac{du}{dx} = \sec 4x \tan 4x \cdot 4 + \sec^2 4x \cdot 4$

$\frac{du}{4(\sec 4x \tan 4x + \sec^2 4x)} = dx$

$= \int \frac{\sec^2 4x + \sec 4x \tan 4x}{u} \cdot \frac{du}{4(\sec 4x \tan 4x + \sec^2 4x)}$

$= \frac{1}{4}\int \frac{1}{u}\,du$

$= \frac{1}{4}\ln|u| + C \longrightarrow \frac{1}{4}\ln|\sec 4x + \tan 4x| + C$

⑧ $\int \frac{\sin x}{\sqrt{\cos x}}\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{du}{-\sin x} = dx$

$= \int \frac{\sin x}{\sqrt{u}} \cdot \frac{du}{-\sin x}$

$= -\int u^{-1/2}\,du$

$= -(2u^{1/2}) + C$

$= -2(\cos x)^{1/2} + C$

or $-2\sqrt{\cos x} + C$

⑨ $\int (\tan x)\ln(\cos x)\,dx$

$u = \ln(\cos x)$

$\frac{du}{dx} = \frac{1}{\cos x} \cdot -\sin x$

$\frac{du}{dx} = -\tan x$

$\frac{du}{-\tan x} = dx$

$= \int \tan x \cdot u \cdot \frac{du}{-\tan x}$

$= -\int u\,du$

$= -\frac{1}{2}u^2 + C$

$= -\frac{1}{2}(\ln(\cos x))^2 + C$

⑩ $\int \frac{1+\cos x}{\sin x}\,dx$

$= \int \frac{1+\cos x}{\sin x} \cdot \frac{1-\cos x}{1-\cos x}\,dx$

$= \int \frac{1-\cos^2 x}{\sin x(1-\cos x)}\,dx$

$= \int \frac{\sin^2 x}{\sin x(1-\cos x)}\,dx$

$= \int \frac{\sin x}{1-\cos x}\,dx$

$u = 1-\cos x$

$\frac{du}{dx} = \sin x$

$\frac{du}{\sin x} = dx$

$= \int \frac{\sin x}{u} \cdot \frac{du}{\sin x}$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|1-\cos x| + C$

⑪ $\int \frac{4}{4x^2+4x+65}\,dx$

$= \int \frac{4}{4(x^2+x+\frac{1}{4})+65-1}\,dx$

$= \int \frac{4}{4(x+\frac{1}{2})^2+64}\,dx$

$= \int \frac{4}{64\left(\frac{(x+\frac{1}{2})^2}{16}+1\right)}\,dx$

$= \frac{1}{16}\int \frac{1}{\left(\frac{x+\frac{1}{2}}{4}\right)^2+1}\,dx$

$= \frac{1}{16}\int \frac{1}{(\frac{1}{4}x+\frac{1}{8})^2+1}\,dx$

$u = \frac{1}{4}x + \frac{1}{8}$

$\frac{du}{dx} = \frac{1}{4}$

$4\,du = dx$

$= \frac{1}{16}\int \frac{1}{u^2+1} \cdot 4\,du$

$= \frac{1}{4}\tan^{-1}(u) + C$

$= \frac{1}{4}\tan^{-1}(\frac{1}{4}x+\frac{1}{8}) + C$

(12) $\int (1+e^x)^2\,dx$

$= \int (1 + 2e^x + e^{2x})\,dx$

$= \int 1\,dx + 2\int e^x\,dx + \int e^{2x}\,dx$

$= x + 2e^x + \int e^u \cdot \frac{du}{2}$

$u = 2x$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$= x + 2e^x + \frac{1}{2}\int e^u\,du$

$= x + 2e^x + \frac{1}{2}e^u + C$

$= x + 2e^x + \frac{1}{2}e^{2x} + C$