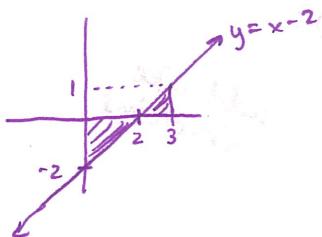


DATE: _____

Fundamental Theorem of Calculus

Using geometry, find: $\int_0^3 (x - 2) dx$



$$\begin{aligned}\int_0^3 (x - 2) dx &= -\frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) \\ &= -2 + \frac{1}{2} \\ &= -\frac{3}{2}\end{aligned}$$

Fundamental Theorem of Calculus

If f is continuous on $[a, b]$ and F is an antiderivative of f on $[a, b]$,

$$\text{then } \int_a^b f(x) dx = F(x) \Big|_a^b$$

$$= F(b) - F(a)$$

(NOTE: FTC also holds true for non-continuous functions, since the Newton-Leibniz Axiom states that f does not need to be continuous, but only that f is Riemann integrable)

Example:

Evaluate: $\int_0^3 (x - 2) dx$

$$\begin{aligned}\int_0^3 (x - 2) dx &= \left(\frac{1}{2}x^2 - 2x \right) \Big|_0^3 \\ &= \frac{1}{2}(3)^2 - 2(3) - \left(\frac{1}{2}(0)^2 - 2(0) \right) \\ &= \frac{9}{2} - 6 - 0 \\ &= \frac{9}{2} - \frac{12}{2} \\ &= -\frac{3}{2}\end{aligned}$$

... same as
in the opening
question

why not $+C$?
 $\left(\frac{1}{2}x^2 - 2x + C \right) \Big|_0^3$
 $\frac{1}{2}(3)^2 - 2(3) + C - \left(\frac{1}{2}(0)^2 - 2(0) + C \right)$
 $\frac{9}{2} - 6 + C - C$
 C cancels out

Evaluate each integral

Example 1: $\int_1^2 (x^2 - 3) dx$

$$\begin{aligned}\int_1^2 (x^2 - 3) dx &= \left(\frac{1}{3}x^3 - 3x\right) \Big|_1^2 \\ &= \frac{1}{3}(2)^3 - 3(2) - \left(\frac{1}{3}(1)^3 - 3(1)\right) \\ &= \frac{8}{3} - 6 - \frac{1}{3} + 3 \\ &= \frac{7}{3} - 3 \\ &= \frac{7}{3} - \frac{9}{3} = \boxed{-\frac{2}{3}}\end{aligned}$$

add like fractions

Example 2: $\int_1^4 \sqrt{x^3} dx$

$$\begin{aligned}\int_1^4 x^{3/2} dx &= \frac{2}{5}x^{5/2} \Big|_1^4 \\ &= \frac{2}{5}(4^{5/2} - 1^{5/2}) \\ &= \frac{2}{5}((\sqrt{4})^5 - 1) \\ &= \frac{2}{5}(2^5 - 1) \\ &= \frac{2}{5}(32 - 1) \\ &= \frac{2}{5}(31) \\ &= \boxed{\frac{62}{5}}\end{aligned}$$

Example 3: $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$

$$\begin{aligned}\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du &= \int_{-2}^{-1} (u - u^{-2}) du \\ &= \left(\frac{1}{2}u^2 - (-u^{-1})\right) \Big|_{-2}^{-1} \\ &= \frac{1}{2}(-1)^2 + (-1)^{-1} - \left(\frac{1}{2}(-2)^2 + (-2)^{-1}\right) \\ &= \frac{1}{2} + -1 - \left(2 - \frac{1}{2}\right) \\ &= \frac{1}{2} + -1 - 2 + \frac{1}{2} \\ &= 2 - 3 \\ &= \boxed{-1}\end{aligned}$$

Example 4: $\int_0^{\pi/4} \sec^2 x \, dx$

$$\begin{aligned}\int_0^{\pi/4} \sec^2 x \, dx &= \tan x \Big|_0^{\pi/4} \\&= \tan \frac{\pi}{4} - \tan 0 \\&= 1 - 0 \\&= \boxed{1}\end{aligned}$$

Example 5: $\int_{-8}^{-1} \frac{x-x^2}{\sqrt[3]{x}} \, dx$

$$\begin{aligned}\int_{-8}^{-1} \frac{x-x^2}{\sqrt[3]{x}} \, dx &= \int_{-8}^{-1} \frac{x-x^2}{x^{1/3}} \, dx \\&= \int_{-8}^{-1} (x-x^2)x^{-1/3} \, dx \\&= \int_{-8}^{-1} (x^{2/3} - x^{5/3}) \, dx \\&= \left(\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3} \right) \Big|_{-8}^{-1} \\&= \frac{3}{5}(-1)^{5/3} - \frac{3}{8}(-1)^{8/3} - \left(\frac{3}{5}(-8)^{5/3} - \frac{3}{8}(-8)^{8/3} \right) \\&= -\frac{3}{5} - \frac{3}{8} - \frac{3}{5}((\sqrt[3]{-8}))^5 + \frac{3}{8}(\sqrt[3]{-8})^8\end{aligned}$$

$= -\frac{3}{5} - \frac{3}{8} - \frac{3}{5}(-32) + \frac{3}{8}(256)$
 $= -\frac{3}{5} - \frac{3}{8} + \frac{96}{5} + \frac{768}{8}$
 $= \frac{93}{5} + \frac{765}{8}$
 $= \frac{4569}{40}$

Example 6: $\int_0^2 |2x-1| \, dx$

$$\begin{aligned}\int_0^2 |2x-1| \, dx &= \int_0^{1/2} -(2x-1) \, dx + \int_{1/2}^2 (2x-1) \, dx \\&= (-x^2 + x) \Big|_0^{1/2} + (x^2 - x) \Big|_{1/2}^2 \\&= -\left(\frac{1}{2}\right)^2 + \frac{1}{2} - (-0^2 + 0) + (2^2 - 2) - \left(\left(\frac{1}{2}\right)^2 - \frac{1}{2}\right) \\&= -\frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2} \\&= -\frac{1}{2} + 1 + 2 \\&= -\frac{1}{2} + 3\end{aligned}$$

$$= \boxed{\frac{5}{2}}$$

