6.3 Antidifferentiation by Parts

$$\frac{d}{dx}(fg) = gf' + fg'$$

$$\frac{d}{dx}(fg) = g\frac{df}{dx} + f\frac{dg}{dx}$$

$$fg = g\frac{df}{\cancel{dx}}\cdot\cancel{dx} + f\frac{dg}{\cancel{dx}}\cdot\cancel{dx}$$

$$fg = \int g\,df + \int f\,dg$$

$$uv = \int v\,du + \int u\,dv$$

$$uv - \int v\,du = \int u\,dv$$

OR $$\int \underbrace{u}\,\underbrace{dv} = \underbrace{uv - \int v\,du}_{\text{formula for integration by parts}}$$

$$\int u\,dv = uv - \int v\,du$$

ex: $\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv} =$

$u = x$ $\qquad \int dv = \int e^x dx$

$\frac{du}{dx} = 1$ $\qquad v = e^x$

$du = dx$

$$\int xe^x dx = \underbrace{x}_{u}\underbrace{e^x}_{v} - \int \underbrace{e^x}_{v}\underbrace{dx}_{du}$$

$$= \boxed{xe^x - e^x + C}$$

ex: $\int \underbrace{x^4}_{\cancel{u}} \underbrace{\ln x \, dx}_{\cancel{dv}}$

$u = x^4$ $\quad \int dv = \cancel{\int \ln x \, dx}$

$\cancel{\frac{du}{dx} = 4x^3}$

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ex: $\int x^4 \underbrace{\ln x}_{u} \, dx$ $\quad$ (x^4 and dx → dv)

$u = \ln x \qquad \int dv = \int x^4 \, dx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{1}{5}x^5$

$du = \frac{1}{x} dx$

$$\begin{aligned}
\int x^4 \ln x \, dx &= \ln x \cdot \frac{1}{5}x^5 - \int \frac{1}{5}x^5 \cdot \frac{1}{x} \, dx \\
&= \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 \, dx \\
&= \frac{1}{5}x^5 \ln x - \frac{1}{5}\left(\frac{1}{5}x^5\right) + C \\
&= \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C
\end{aligned}$$

ex: $\int \underbrace{x}_{u}\,\underbrace{\cos 3x\,dx}_{dv}$

$u = x \qquad \int dv = \int \cos 3x\,dx$

$\frac{du}{dx} = 1 \qquad v = \frac{1}{3}\sin 3x$

$du = dx$

$w = 3x$

$\frac{dw}{dx} = 3$

$\frac{dw}{3} = dx$

$\int \cos w \cdot \frac{dw}{3}$

$\frac{1}{3}\sin w$

$\int \sin w \cdot \frac{dw}{3}$

$-\frac{1}{3}\cos w$

$$\int x\cos 3x\,dx = x\cdot\frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,dx$$

$$= \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$$

$$= \frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x\right) + C$$

$$= \boxed{\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C}$$

ex: $\int \underbrace{x^2}_{u} \underbrace{e^x dx}_{dv}$

$u = x^2 \qquad \int dv = \int e^x dx$

$\frac{du}{dx} = 2x \qquad v = e^x$

$du = 2x\,dx$

$$\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x\,dx$$

$$= x^2 e^x - 2\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv}$$

$u = x \qquad \int dv = \int e^x dx$

$\frac{du}{dx} = 1 \qquad v = e^x$

$du = dx$

$$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2\int e^x dx$$

$$= \boxed{x^2 e^x - 2xe^x + 2e^x + C}$$

ex: $\int \underbrace{e^x}_{u} \underbrace{\sin x\,dx}_{dv}$

$u = e^x \qquad \int dv = \int \sin x\,dx$

$\frac{du}{dx} = e^x \qquad v = -\cos x$

$du = e^x dx$

$$\int e^x \sin x\,dx = e^x(-\cos x) - \int -\cos x \cdot e^x\,dx$$

$$= -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x\,dx}_{dv}$$

$u = e^x \qquad \int dv = \int \cos x\,dx$

$\frac{du}{dx} = e^x \qquad v = \sin x$

$du = e^x dx$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int \sin x \cdot e^x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$+\int e^x \sin x\,dx \qquad\qquad +\int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \boxed{\tfrac{1}{2}\left(-e^x \cos x + e^x \sin x\right)}$$