

### 6.3 Parametric Equations

Target 10A: Define and Graph Parametric Equations  
Target 10B: Perform Parametric/Rectangular Conversions

Think about this: Imagine a baseball is thrown across a field. What are the variables involved?

#### Parametric Equations

- $x(t)$  → horizontal distance in terms of time
- $y(t)$  → vertical distance in terms of time

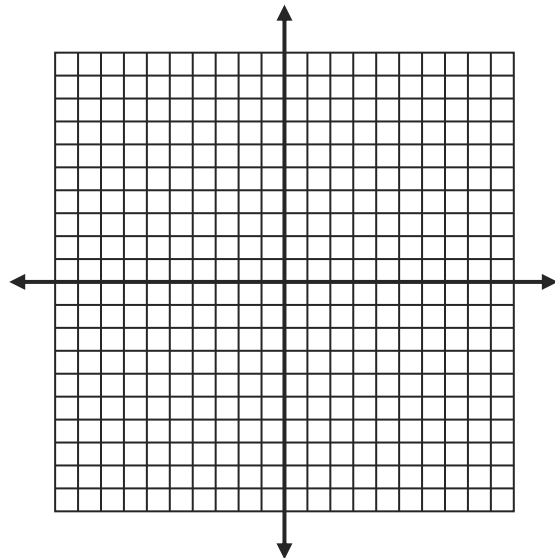
#### Graphing Parametric Equations

Construct a table choosing values of  $t$  and then determine the values of  $x$  and  $y$ .

*Example 1:* Make a table of values and sketch the curve, indicating the direction of your graph.

$$x(t) = t^2 - 2, y(t) = 3t$$

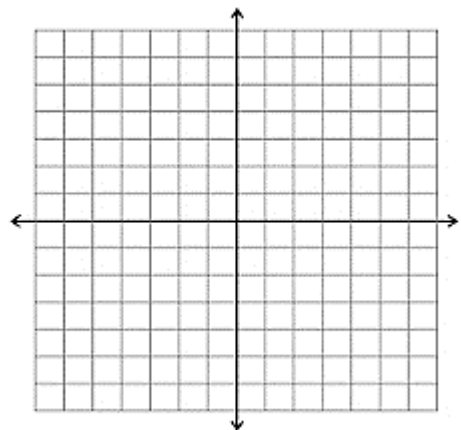
$t$	$x$	$y$
-3		
-2		
-1		
0		
1		
2		
3		



*Example 2:* Make a table of values and sketch the curve, indicating the direction of your graph.

$$\begin{aligned} x(t) &= 5 - 3t, \\ y(t) &= 2 + t, \end{aligned} \quad -1 \leq t \leq 3$$

$t$	$x$	$y$



*Example 3:* ☒ Using your graphing calculator, graph the parametric equations:

$$x(t) = 4t^3 - 16t^2 + 15t, \quad y(t) = 2$$

$$0 \leq t \leq 5, \quad tstep = 0.05$$

Window:  $X[-4,6], Y[-3,6], Xscale: 1, Yscale: 1$

When and where does the particle reverse direction?

### Converting from Parametric to Rectangular

Determining a Rectangular Equation for Given Parametric Equations

- Solve either equation for  $t$ .
- Substitute that value of  $t$  into the other equation.
- Calculate the restrictions on the variables  $x$  and  $y$  based on the restrictions on  $t$ .

*Example:* Write the parametric equations in rectangular form.

$$x(t) = \sqrt{t-2}, \quad y(t) = 3t$$

*Solution:*

$$x = \sqrt{t-2}$$

$$x^2 = t - 2$$

$$x^2 + 2 = t$$

$$y = 3t$$

$$y = 3(x^2 + 2)$$

$$y = 3x^2 + 6$$

*Steps:*

Write one of the parametric equations and then solve for  $t$ .

Write the other parametric equation.

Substitute in expression for  $t$ .

Simplify.

Restriction on  $t$ :

Since  $x(t) = \sqrt{t-2}$  & can't  $\sqrt{\quad}$  a negative #,  $t - 2$  needs to be  $\geq$  zero.

$$\therefore, t - 2 \geq 0, \text{ so } t \geq 2$$

If  $t \geq 2$ , then  $x \geq 0$  and  $y \geq 6$

*ANSWER:*

$$y = 3x^2 + 6, \text{ where } x \geq 0 \text{ and } y \geq 6$$

Now you try, write the parametric equations in rectangular form.

1.  $x(t) = \frac{1}{2}t + 4, \quad y(t) = t^3$

2.  $x(t) = 5 - 3t, \quad y(t) = 2 + t, \quad -1 \leq t \leq 3$

3.  $x(t) = \frac{1}{t-3}, \quad y(t) = 4t + 5$

**More Practice**

**Parametric Equations**

<http://tutorial.math.lamar.edu/Classes/CalcII/ParametricEqn.aspx>

<https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1>

<https://people.richland.edu/james/lecture/m116/conics/parametric.html>

[https://www.varsitytutors.com/hotmath/hotmath\\_help/topics/parametric-equations](https://www.varsitytutors.com/hotmath/hotmath_help/topics/parametric-equations)

<https://youtu.e/9kKZHQtYP7g>

<https://youtu.be/tsnHL1Lb5MU>

<https://youtu.be/ITOSsIFAI18>

**Homework Assignment**

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