

Unit 10 (Chapter 6): Parametric & Polar

6.3 Parametric Equations

Target 10A: Define and Graph Parametric Equations
Target 10B: Perform Parametric/Rectangular Conversions

Think about this: Imagine a baseball is thrown across a field. What are the variables involved?

Parametric Equations

- $x(t)$ → horizontal distance in terms of time
- $y(t)$ → vertical distance in terms of time

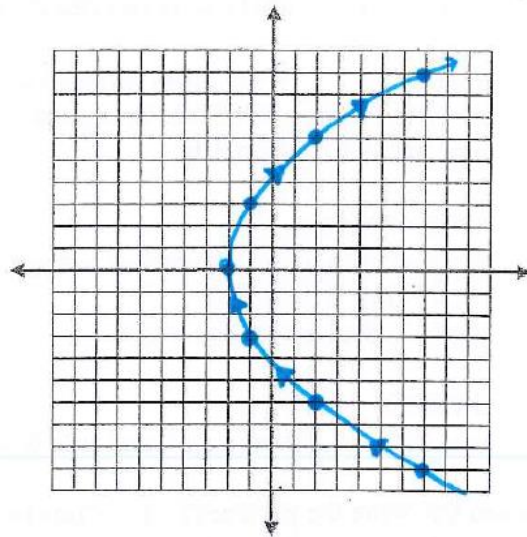
Graphing Parametric Equations

Construct a table choosing values of t and then determine the values of x and y .

Example 1: Make a table of values and sketch the curve, indicating the direction of your graph.

$$x(t) = t^2 - 2, y(t) = 3t$$

t	x	y
-3	7	-9
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3
2	2	6
3	7	9

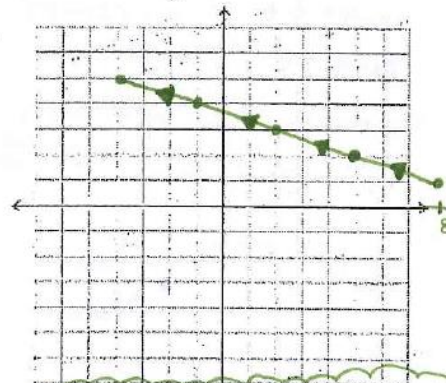


Example 2: Make a table of values and sketch the curve, indicating the direction of your graph.

$$x(t) = 5 - 3t, y(t) = 2 + t$$

$-1 \leq t \leq 3$
only these times

t	x	y
-1	8	1
0	5	2
1	2	3
2	-1	4
3	-4	5



notice: the picture/graph is a segment

Example 3: Using your graphing calculator, graph the parametric equations:

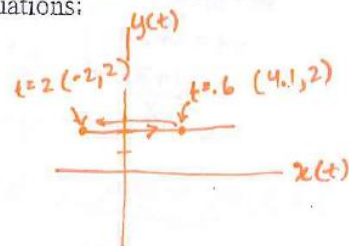
$$x(t) = 4t^3 - 16t^2 + 15t, y(t) = 2$$

$0 \leq t \leq 5, tstep = 0.05$
Window: $X[-4,6], Y[-3,6], Xscale: 1, Yscale: 1$

When and where does the particle reverse direction?

time pt.

TRACE...



@ $t = 0.6$ particle is @ $(4.1, 2)$
@ $t = 2$ particle is @ $(-2, 2)$ } this is where + when particle changes direction

MENU
Graph entry
Parametric
Window/Edit
Window Settings

Converting from Parametric to Rectangular

Determining a Rectangular Equation for Given Parametric Equations

- Solve either equation for t .
- Substitute that value of t into the other equation.
- Calculate the restrictions on the variables x and y based on the restrictions on t .

Example: Write the parametric equations in rectangular form.

$$x(t) = \sqrt{t-2}, \quad y(t) = 3t$$

Solution:

$$x = \sqrt{t-2}$$

$$x^2 = t - 2$$

$$x^2 + 2 = t$$

$$y = 3t$$

$$y = 3(x^2 + 2)$$

$$y = 3x^2 + 6$$

Steps:

Write one of the parametric equations and then solve for t .

Write the other parametric equation.

Substitute in expression for t .

Simplify.

Restriction on t :

Since $x(t) = \sqrt{t-2}$ & can't $\sqrt{\text{a negative \#}}$, $t-2$ needs to be \geq zero.

$$\therefore, t - 2 \geq 0, \text{ so } t \geq 2$$

If $t \geq 2$, then $x \geq 0$ and $y \geq 6$

ANSWER:

$$y = 3x^2 + 6, \text{ where } x \geq 0 \text{ and } y \geq 6$$

Now you try, write the parametric equations in rectangular form.

1. $x(t) = \frac{1}{2}t + 4, \quad y(t) = t^3$

$$\begin{aligned} x &= \frac{1}{2}t + 4 & y &= t^3 \\ x - 4 &= \frac{1}{2}t & y &= (2x - 8)^3 \\ 2x - 8 &= t \end{aligned}$$

no restrictions on t (no $\sqrt{\quad}$, no denominator etc)
 \therefore , no restrictions on x or y .

$$\boxed{y = (2x - 8)^3 \quad \forall x \text{ and } \forall y}$$

"for all"

3. $x(t) = \frac{1}{t-3}, \quad y(t) = 4t + 5$

$$\begin{aligned} x &= \frac{1}{t-3} & y &= 4t + 5 \\ x(t-3) &= 1 & y &= 4\left(\frac{1+3x}{x}\right) + 5 \\ x^2 - 3x &= 1 & y &= \frac{4 + 12x}{x} + 5 \\ x + t &= 1 + 3x & y &= \frac{4}{x} + 12 + 5 \\ t &= \frac{1+3x}{x} & y &= \frac{4}{x} + 17 \end{aligned}$$

$t \neq 3$, b/c $t-3 \neq 0$ (denominator)

$$\therefore, y(3) \neq 17$$

$$\boxed{y = \frac{4}{x} + 17 \text{ where } y \neq 17}$$

2. $x(t) = 5 - 3t, \quad y(t) = 2 + t, \quad -1 \leq t \leq 3$

$$\begin{aligned} x &= 5 - 3t & y &= 2 + t \\ x - 5 &= -3t & y &= 2 + \frac{x-5}{-3} \\ \frac{x-5}{-3} &= t & y &= 2 + \frac{-x+5}{3} \\ & & y &= -\frac{x}{3} + \frac{11}{3} \end{aligned}$$

$$x(-1) = 8$$

$$x(3) = -4$$

$$\therefore, -4 \leq x \leq 8$$

$$y(-1) = 1$$

$$y(3) = 5$$

$$\therefore, 1 \leq y \leq 5$$

$$\boxed{y = -\frac{x}{3} + \frac{11}{3} \quad \text{where } -4 \leq x \leq 8 \text{ and } 1 \leq y \leq 5}$$

More Practice

Parametric Equations

<http://tutorial.math.lamar.edu/Classes/CalcII/ParametricEqn.aspx>

<https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1>

<https://people.richland.edu/james/lecture/m116/conics/parametric.html>

https://www.varsitytutors.com/hotmath/hotmath_help/topics/parametric-equations

<https://youtu.e/9kKZHQtYP7g>

<https://youtu.be/tsnHL1Lb5MU>

<https://youtu.be/ITOSsIFAI18>

Homework Assignment

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