

Unit 10 (Chapter 6): Parametric & Polar

6.3 Parametric Equations Practice

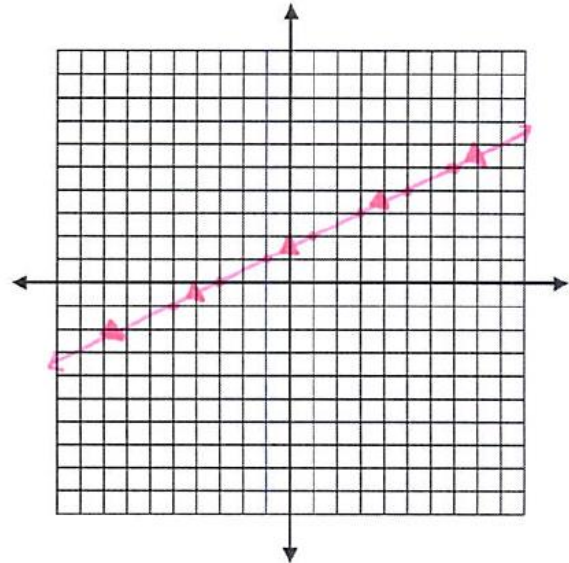
Target 10A: Define and Graph Parametric Equations

Target 10B: Perform Parametric/Rectangular Conversions

1. a) Make a table of values and sketch the curve, indicating the direction of your graph.

$$x(t) = 1 - 2t, y(t) = 2 - t$$

t	x	y
-3	7	5
-2	5	4
-1	3	3
0	1	2
1	-1	1
2	-3	0
3	-5	-1



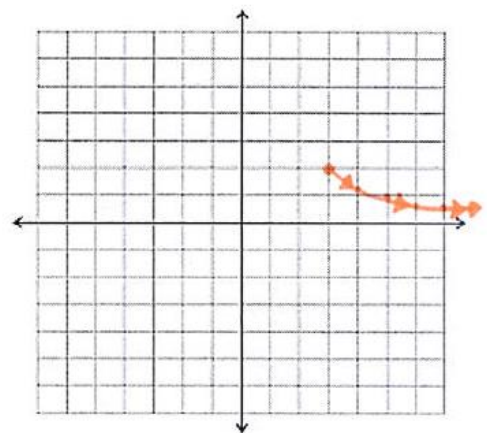
b) Eliminate the parameter of the equations above and give the resulting rectangular equation.

$$\begin{aligned}
 x &= 1 - 2t \\
 x - 1 &= -2t \\
 \frac{x-1}{-2} &= t
 \end{aligned}
 \quad
 \begin{aligned}
 y &= 2 - t \\
 y &= 2 - \left(\frac{x-1}{-2}\right) \\
 y &= 2 + \frac{x-1}{2}
 \end{aligned}
 \quad
 \begin{aligned}
 y &= 2 + \frac{x}{2} - \frac{1}{2} \\
 y &= \frac{x}{2} + \frac{3}{2} \quad \text{or} \quad \boxed{y = \frac{1}{2}x + \frac{3}{2}}
 \end{aligned}$$

2. a) Make a table of values and sketch the curve, indicating the direction of your graph.

$$\begin{aligned}
 x(t) &= t + 2, & t &\geq 2 \\
 y(t) &= \frac{4}{t}
 \end{aligned}$$

t	x	y
2	4	2
3	5	$\frac{4}{3} \approx 1.333$
4	6	1
5	7	$\frac{4}{5} = 0.8$
6	8	$\frac{2}{3} \approx 0.667$



b) Eliminate the parameter of the equations above and give the resulting rectangular equation.

$$\begin{aligned}
 x &= t + 2 \\
 x - 2 &= t
 \end{aligned}
 \quad
 \begin{aligned}
 y &= \frac{4}{t} \\
 \boxed{y} &= \frac{4}{x-2}
 \end{aligned}$$

3. ☒ Using your graphing calculator, graph the parametric equations:

$$x(t) = \cos t, \quad y(t) = \sin t$$

What does the graph look like? *Circle*

4. ☒ Using your graphing calculator, graph the parametric equations:

$$x(t) = 2\sin t, \quad y(t) = 4\cos t$$

What does the graph look like? *Ellipse*

Converting from Parametric Trig Equations to Rectangular

Determining a Rectangular Equation for Given Parametric Trig Equations

- Use a trig identity to help convert (i. e., $\sin^2 \theta + \cos^2 \theta = 1$)
- Solve the parametric equations for the trig functions in the trig identity & sub into trig identity.
- Rewrite the rectangular equation into a recognizable equation.

Example 1: Write the parametric equations in rectangular form.

$$x(t) = \frac{1}{2} \cos t, \quad y(t) = \frac{1}{2} \sin t$$

$$\begin{aligned} x &= \frac{1}{2} \cos t & y &= \frac{1}{2} \sin t \\ 2x &= \cos t & 2y &= \sin t \end{aligned}$$

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ (2y)^2 + (2x)^2 &= 1 \\ 4y^2 + 4x^2 &= 1 \\ y^2 + x^2 &= \frac{1}{4} \end{aligned}$$

... 😊
circle centered @ (0,0)
w/ radius = $\frac{1}{2}$

Example 2: Write the parametric equations in rectangular form.

$$x(t) = 2 \sin t, \quad y(t) = 4 \cos t$$

$$\begin{aligned} x &= 2 \sin t & y &= 4 \cos t \\ \frac{x}{2} &= \sin t & \frac{y}{4} &= \cos t \end{aligned}$$

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{4}\right)^2 &= 1 \\ \frac{x^2}{4} + \frac{y^2}{16} &= 1 \end{aligned}$$

... 😊
ellipse centered @ (0,0)
where $a=4$, $b=2$,
major axis is vertical

Example 3: Write the parametric equations in rectangular form.

$$x(t) = 3 \cos t - 1, \quad y(t) = 2 \sin t + 4$$

$$\begin{aligned} x &= 3 \cos t - 1 & y &= 2 \sin t + 4 \\ x+1 &= 3 \cos t & y-4 &= 2 \sin t \\ \frac{x+1}{3} &= \cos t & \frac{y-4}{2} &= \sin t \end{aligned}$$

$$\begin{aligned} \sin^2 t + \cos^2 t &= 1 \\ \left(\frac{y-4}{2}\right)^2 + \left(\frac{x+1}{3}\right)^2 &= 1 \\ \frac{(y-4)^2}{4} + \frac{(x+1)^2}{9} &= 1 \end{aligned}$$

... 😊
ellipse centered
@ (-1, 4)
where $a=3$, $b=2$
major axis is
horizontal

More Practice

Parametric Equations

<http://tutorial.math.lamar.edu/Classes/CalcII/ParametricEqn.aspx>

<https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1>

<https://people.richland.edu/james/lecture/m116/conics/parametric.html>

https://www.varsitytutors.com/hotmath/hotmath_help/topics/parametric-equations

<https://youtu.e/9kKZHQtYP7g>

<https://youtu.be/tsnHL1Lb5MU>

<https://youtu.be/ITOSsIFAI18>

Homework Assignment

p.530 #3,5,11,15,19,23,25,61