DATE: ______________

Integration by Parts

1. $\int x\cos 5x\,dx$

$u = x \qquad \int dv = \int \cos 5x\,dx$

$\frac{du}{dx} = 1 \qquad v = \frac{1}{5}\sin 5x$

$du = dx$

$\int x\cos 5x\,dx = x\cdot\frac{1}{5}\sin 5x - \int \frac{1}{5}\sin 5x\cdot dx$

$= \frac{1}{5}x\sin 5x - \frac{1}{5}\left(-\frac{1}{5}\cos 5x\right) + C$

$= \boxed{\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C}$

2. $\int \ln(2x+1)\,dx$

$u = \ln(2x+1) \qquad \int dv = \int dx$

$\frac{du}{dx} = \frac{1}{2x+1}\cdot 2 \qquad v = x$

$du = \frac{2}{2x+1}\,dx$

$\int \ln(2x+1)\,dx = \ln(2x+1)\cdot x - \int x\cdot\frac{2}{2x+1}\,dx$

$= x\ln(2x+1) - \int \frac{2x}{2x+1}\,dx$

$= x\ln(2x+1) - \int \frac{w-1}{w}\cdot\frac{dw}{2}$

$w = 2x+1$

$\frac{dw}{dx} = 2$

$\frac{dw}{2} = dx$

$w = 2x+1$

$w-1 = 2x$

$= x\ln(2x+1) - \frac{1}{2}\int\left(1-\frac{1}{w}\right)dw$

$= x\ln(2x+1) - \frac{1}{2}w + \frac{1}{2}\ln|w| + C$

$= \boxed{x\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln|2x+1| + C}$

3. $\int (\ln x)^2\,dx$

$u = (\ln x)^2 \qquad \int dv = \int dx$

$\frac{du}{dx} = 2(\ln x)\frac{1}{x} \qquad v = x$

$du = 2(\ln x)\frac{1}{x}\,dx$

$\int (\ln x)^2\,dx = (\ln x)^2\cdot x - \int x\cdot 2(\ln x)\frac{1}{x}\,dx$

$= x(\ln x)^2 - 2\int \ln x\,dx$

$u = \ln x \qquad \int dv = \int dx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = x$

$du = \frac{1}{x}dx$

$= x(\ln x)^2 - 2\left((\ln x)(x) - \int x\,\frac{1}{x}\,dx\right)$

$= x(\ln x)^2 - 2x\ln x + 2\int dx$

$= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C}$

4. $\int e^{2x}\sin 3x\,dx$

$u = e^{2x} \qquad dv = \sin 3x\,dx$

$\frac{du}{dx} = 2e^{2x} \qquad v = -\frac{1}{3}\cos 3x$

$du = 2e^{2x}dx$

$\int e^{2x}\sin 3x\,dx = e^{2x}\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\cdot 2e^{2x}\,dx$

$= -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\int e^{2x}\cos 3x\,dx$

$u = e^{2x} \qquad \int dv = \int \cos 3x\,dx$

$du = 2e^{2x}dx \qquad v = \frac{1}{3}\sin 3x$

$\int e^{2x}\sin 3x\,dx = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\left(e^{2x}\cdot\frac{1}{3}\sin 3x - \int\frac{1}{3}\sin 3x\right.$ $\left. 2e^{2x}dx\right)$

$\int e^{2x}\sin 3x\,dx = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}\int e^{2x}\sin 3x\,dx$

$\frac{13}{9}\int e^{2x}\sin 3x\,dx = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{9}e^{2x}\cos 3x$

$\int e^{2x}\sin 3x\,dx = \boxed{-\frac{3}{13}e^{2x}\cos 3x + \frac{2}{13}e^{2x}\cos 3x + C}$

5. $\int \frac{\ln x}{x^2}\,dx$

$u = \ln x \qquad \int dv = \int x^{-2}dx$

$du = \frac{1}{x}dx \qquad v = -\frac{1}{x}$

$\int \frac{\ln x}{x^2}\,dx = \ln x\cdot -\frac{1}{x} - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx$

$= -\frac{1}{x}\ln x + \int x^{-2}\,dx$

$= \boxed{-\frac{\ln x}{x} - \frac{1}{x} + C}$

6. $\int \frac{x}{e^{2x}}\,dx$

$= \int xe^{-2x}\,dx$

$u = x \qquad dv = e^{-2x}dx$

$du = dx \qquad v = -\frac{1}{2}e^{-2x}$

$\int \frac{x}{e^{2x}}\,dx = x\cdot -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx$

$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\left(-\frac{1}{2}e^{-2x}\right) + C$

$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$