DATE: ________________

Integration by Parts Practice

1. The integral $\int \frac{1}{x \ln x}\, dx$ can be found by:

$\boxed{\text{(a) making the substitution } u = \ln x}$

(b) making the substitution $u = \frac{1}{x}$

(c) using integration by parts, with $u = \ln x$ and $dv = x\, dx$

(d) taking the reciprocal of $\int x \ln x\, dx$

(e) none of the above

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2. The integral $\int x \sin x\, dx$ can be found by:

(a) making the substitution $u = x$

(b) making the substitution $u = \sin x$

(c) using integration by parts, with $u = \sin x$and $dv = x\, dx$

$\boxed{\text{(d) using integration by parts, with } u = x \text{ and dv } dv = \sin x\, dx}$

(e) none of the above

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3. $\int x^2 \ln x\, dx$

$u = \ln x$ $\qquad$ $\int dv = \int x^2\, dx$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad$ $v = \frac{1}{3}x^3$

$du = \frac{1}{x}\, dx$

$\int x^2 \ln x\, dx = \ln x\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\, dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right) + C$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

4. $\int x^3 \cos x\, dx$

	$\underline{u}$	$\underline{dv}$
+	x^3	$\cos x$
−	$3x^2$	$\sin x$
+	$6x$	$-\cos x$
−	6	$-\sin x$
+	0	$\cos x$

$\int x^3 \cos x\, dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + C$

5. $\int_0^1 (7-3x)e^{6x}\,dx$

	u	dv
$+$	$7-3x$	e^{6x}
$-$	-3	$\frac{1}{6}e^{6x}$
$+$	0	$\frac{1}{36}e^{6x}$

$$\int_0^1 (7-3x)e^{6x}\,dx = \left((7-3x)\tfrac{1}{6}e^{6x} + \tfrac{1}{12}e^{6x}\right)\Big|_0^1$$

$$= (7-3(1))\tfrac{1}{6}e^{6(1)} + \tfrac{1}{12}e^{6(1)} - \left((7-3(0))\tfrac{1}{6}e^{0} + \tfrac{1}{12}e^{0}\right)$$

$$= \tfrac{2}{3}e^{6} + \tfrac{1}{12}e^{6} - \tfrac{7}{6} - \tfrac{1}{12} = \boxed{\tfrac{3}{4}e^{6} - \tfrac{5}{4}}$$

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6. $\int e^{3x}\cos x\,dx$

$u = e^{3x}$ $\quad \int dv = \int \cos x\,dx$

$\frac{du}{dx} = 3e^{3x}$ $\quad v = \sin x$

$du = 3e^{3x}dx$

$$\int e^{3x}\cos x\,dx = e^{3x}\cdot \sin x - \int \sin x \cdot 3e^{3x}dx$$

$u = e^{3x}$ $\quad \int dv = \int \sin x\,dx$

$\frac{du}{dx} = 3e^{3x}$ $\quad v = -\cos x$

$du = 3e^{3x}dx$

$$\int e^{3x}\cos x\,dx = e^{3x}\sin x - 3\left(e^{3x}\cdot -\cos x - \int -\cos x \cdot 3e^{3x}dx\right)$$

$$\int e^{3x}\cos x\,dx = e^{3x}\sin x + 3e^{3x}\cos x - 9\int e^{3x}\cos x\,dx$$

$$10\int e^{3x}\cos x\,dx = e^{3x}\sin x + 3e^{3x}\cos x \longrightarrow \int e^{3x}\cos x\,dx = \boxed{\tfrac{1}{10}\left(e^{3x}\sin x + 3e^{3x}\cos x\right) + C}$$

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7. The function f is continuous and $f(0) = 1, f(2) = 5$, and $\int_0^2 f(x)dx = 3$. Find $\int_0^2 x\,f'(x)\,dx$

$$\int_0^2 x f'(x)\,dx$$

$u = x$ $\quad \int dv = \int f'(x)dx$

$\frac{du}{dx} = 1$ $\quad v = f(x)$

$du = dx$

$$\int_0^2 x f'(x)dx = \left[x\cdot f(x)\right]\Big|_0^2 - \int_0^2 f(x)\,dx$$

$$= 2f(2) - 0f(0) - \int_0^2 f(x)dx$$

$$= 2(5) - 0 - 3$$

$$= 10 - 3$$

$$= \boxed{7}$$