6.4 Solve the differential equation.

ex: $\frac{dy}{dx} = \frac{x}{y^2}$

$dy = \frac{x}{y^2}\,dx$ need y's w/ dy's and x's w/ dx's

$y^2\,dy = x\,dx$

$\int y^2\,dy = \int x\,dx$

$\frac{1}{3}y^3 = \frac{1}{2}x^2 + C$

$y^3 = \frac{3}{2}x^2 + C_2$ where $C_2 = 3C$

$y = \sqrt[3]{\frac{3}{2}x^2 + C_2}$

$y = \sqrt[3]{\frac{3}{2}x^2 + C}$ where C is some constant

Ex: $\frac{dy}{dx} = 2xy + 2x$ @ $(1, 2)$ (x, y)

$dy = (2xy + 2x)\,dx$

$dy = 2x(y+1)\,dx$

$\frac{1}{y+1}\,dy = 2x\,dx$ — y's w/ dy's, x's w/ dx's

$\int \frac{1}{y+1}\,dy = \int 2x\,dx$

$u = y+1$, $du = 1$, $dy = du$, $\int \frac{1}{u}\,du$

$\ln|y+1| = x^2 + C$ ← general solution; need particular solution

$\ln|2+1| = 1^2 + C$

$\ln 3 = 1 + C$

$-1 + \ln 3 = C$

$\ln|y+1| = x^2 - 1 + \ln 3$ solve for y.

$e^{\ln|y+1|} = e^{x^2 - 1 + \ln 3}$

$|y+1| = e^{x^2-1} \cdot e^{\ln 3}$

$|y+1| = 3e^{x^2-1}$

$y+1 = \pm 3e^{x^2-1}$ — keep pos. b/c $y+1 > 0$ when $y = 2$

$\boxed{y = 3e^{x^2-1} - 1}$

ex: $\frac{dy}{dx} = e^{x-2y}$ @ $(0,0)$

$dy = e^{x-2y}\,dx$

$dy = e^x \cdot e^{-2y}\,dx$

$dy = \frac{e^x}{e^{2y}}\,dx$

$\int e^{2y}\,dy = \int e^x\,dx$

$u = 2y$

$\frac{du}{dy} = 2$

$\frac{du}{2} = dy$

$\int e^u \cdot \frac{du}{2} = e^x + C$

$\frac{1}{2}\int e^u\,du = e^x + C$

$\frac{1}{2}e^u = e^x + C$

$\frac{1}{2}e^{2y} = e^x + C$ general solution, need particular solution @ $(0,0)$

$\frac{1}{2}e^{2(0)} = e^0 + C$

$\frac{1}{2}e^0 = 1 + C$

$\frac{1}{2} = 1 + C$

$-\frac{1}{2} = C$

$\frac{1}{2}e^{2y} = e^x - \frac{1}{2}$ solve for y.

$\ln e^{2y} = \ln 2e^x - 1$

$2y = \ln|2e^x - 1|$

$y = \frac{1}{2}\ln|2e^x - 1|$ drop abs value b/c when $x = 0$, $2e^x - 1 > 0$

$\boxed{y = \frac{1}{2}\ln(2e^x - 1)}$