Look over the work from 3 different students for the problem listed below.
For each student, if there are any errors, identify where any errors occur and correct the work.

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

    (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

Work for problem 6(a)

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$$

$$e^{2y}\, dy = 3x^2\, dx$$

$$\int e^{2y}\, dy = \int 3x^2\, dx$$

$$\frac{1}{2}e^{2y} = x^3 + C \qquad f(0) = \frac{1}{2}$$
$$\qquad\qquad\qquad\qquad x=0$$
$$\qquad\qquad\qquad\qquad y=\frac{1}{2}$$

$$\frac{1}{2}\cdot e^1 = 0 + C$$

$$C = \frac{e}{2}$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{e}{2}$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln\left(2x^3 + e\right)$$

$$y = \frac{\ln(2x^3+e)}{2}$$

$$\therefore y = \frac{\ln(2x^3+e)}{2}$$

Work for problem 6(a)

$$e^{2y}\,dy = 3x^2\,dx$$

$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{e^{2y}}{2} = x^3$$

$$e^{2y} = 2x^3$$

$$2y = \ln 2x^3$$

$$y = \frac{\ln 2x^3}{2}$$

$$f(x) = \frac{\ln 2x^3}{2}$$

$$f(0) = \frac{\ln 0}{2} = \frac{1}{2}$$

Work for problem 6(a)

$$f(x) = \sqrt{\frac{3x^2}{e^{2y}}\frac{dy}{dx}}$$

$$= \int e^{2y}\,dy = \int 3x^2\,dx$$

$$= 2e^{2y} = x^3 + C$$

but @ $x=0, y=\frac{1}{2}$

AND $2e^{2(\frac{1}{2})} = 0^3 + C$

$$2e = C$$

$$2e^{2y} = x^3 + 2e$$

$$e^{2y} = \frac{x^3 + 2e}{2}$$

$$2y = \ln\left(\frac{x^3 + 2e}{2}\right)$$

$$y = \frac{1}{2}\ln\left(\frac{x^3 + 2e}{2}\right)$$