Look over the work from 3 different students for the problem listed below.
For each student, if there are any errors, identify where any errors occur and correct the work.

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.

   (a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.

Work for problem 6(a)

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$$

$$e^{2y}\, dy = 3x^2\, dx$$

$$\int e^{2y}\, dy = \int 3x^2\, dx$$

$$\frac{1}{2}e^{2y} = x^3 + C \qquad f(0)=\frac{1}{2}$$
$$\tiny x=0 \quad y=\frac{1}{2}$$

$$\frac{1}{2}\cdot e^1 = 0 + C$$

$$C = \frac{e}{2}$$

$$\frac{1}{2}e^{2y} = x^3 + \frac{e}{2}$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln\left(2x^3 + e\right)$$

$$y = \frac{\ln\left(2x^3 + e\right)}{2} \quad \leftarrow \text{absolute value}$$

$$\therefore\ y = \frac{\ln\left(2x^3 + e\right)}{2}$$

Work for problem 6(a)

$e^{2y} dy = 3x^2 dx$

$\int e^{2y} dy = \int 3x^2 dx$

$\dfrac{e^{2y}}{2} = x^3$ ← missing $+C$

$e^{2y} = 2x^3$

$2y = \ln 2x^3$

$y = \dfrac{\ln 2x^3}{2}$

$f(x) = \dfrac{\ln 2x^3}{2}$

$f(0) = \dfrac{\ln 0}{2} = \dfrac{1}{2}$

Work for problem 6(a)

inaccurate notation

$f(x) = \int \sqrt{\dfrac{3x^2}{e^{2y}}} \ \dfrac{dy}{dx}$

$= \int e^{2y} dy = \int 3x^2 dx$

antiderive incorrectly

$\frac{1}{2} 2e^{2y} = x^3 + C$

but @ $x = 0,\ y = \frac{1}{2}$

AND $2e^{2(\frac{1}{2})} = 0^3 + C$

$2e = C$

$2e^{2y} = x^3 + 2e$

$e^{2y} = \dfrac{x^3 + 2e}{2}$

$2y = \ln\left(\dfrac{x^3 + 2e}{2}\right)$

$y = \frac{1}{2} \ln\left(\dfrac{x^3 + 2e}{2}\right)$