

6.3 Polar Coordinates

Target 10D: Understand the Polar Coordinate System by performing Polar/Rectangular Coordinate Conversions

Review of Prior Concepts

1. Find the magnitude of the vector $\langle \sqrt{3}, 1 \rangle$.

$$\begin{aligned} \text{magnitude} &= \sqrt{(\sqrt{3})^2 + 1^2} \\ &= \sqrt{3+1} \\ &= \sqrt{4} \\ &= \boxed{2} \end{aligned}$$

2. Find the direction angle of the vector $\langle \sqrt{3}, 1 \rangle$.

$$\begin{aligned} \cos \theta &= \frac{\sqrt{3}}{2} && \text{or } \sin \theta = \frac{1}{2} \\ \theta &= \frac{\pi}{6} \end{aligned}$$

Polar Coordinates

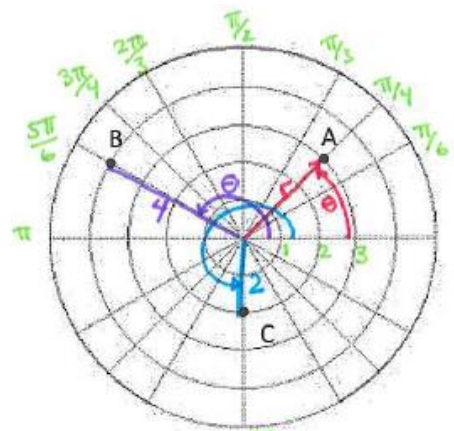
(r, θ)

The polar coordinates of point A is $(3, 45^\circ)$ or $(3, \frac{\pi}{4})$.

Example 1: Identify the coordinates of:

a) point B $(4, 5\pi/6)$

b) point C $(2, 3\pi/2)$

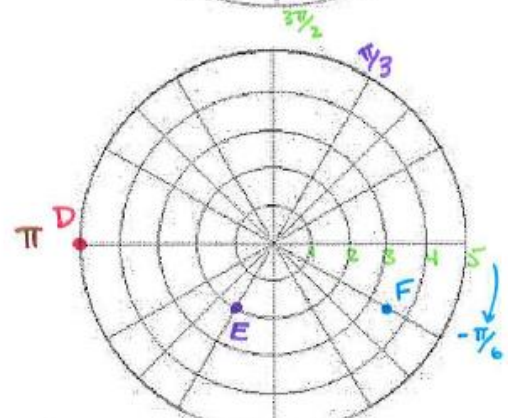


Example 2: Plot each of the given points:

a) D $(5, \pi)$

b) E $(-2, \frac{\pi}{3})$

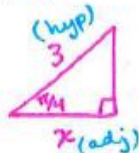
c) F $(3.5, -\frac{\pi}{6})$



Example 3:

Determine the rectangular (x, y) coordinates of point A.

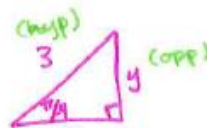
A $(3, \frac{\pi}{4})$



$$\cos \frac{\pi}{4} = \frac{x}{3}$$

$$\frac{\sqrt{2}}{2} = \frac{x}{3}$$

$$\frac{3\sqrt{2}}{2} = x$$

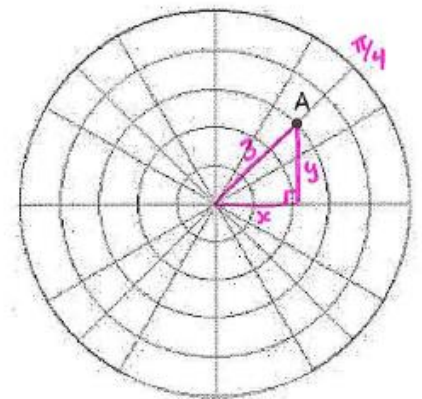


$$\sin \frac{\pi}{4} = \frac{y}{3}$$

$$\frac{\sqrt{2}}{2} = \frac{y}{3}$$

$$\frac{3\sqrt{2}}{2} = y$$

$$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \right)$$



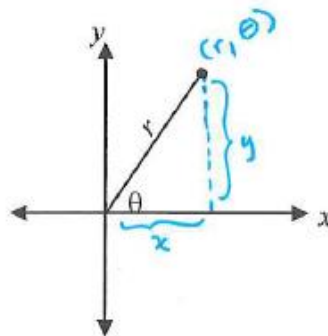
Convert Polar Coordinates to Rectangular Coordinates

$$(r, \theta) \rightarrow (x, y)$$

$$\cos \theta = \frac{x}{r} \quad \sin \theta = \frac{y}{r}$$

$$r \cos \theta = x \quad r \sin \theta = y$$

$$(x, y) = (r \cos \theta, r \sin \theta)$$



Example 4:

Convert $(2, \frac{5\pi}{6})$ to rectangular coordinates.

$$\begin{matrix} r & \rightarrow & \theta \\ \uparrow & & \downarrow \end{matrix}$$

$$(2 \cos \frac{5\pi}{6}, 2 \sin \frac{5\pi}{6})$$

$$(2(-\frac{\sqrt{3}}{2}), 2(\frac{1}{2}))$$

$$\boxed{(-\sqrt{3}, 1)}$$

Convert Rectangular Coordinates to Polar Coordinates

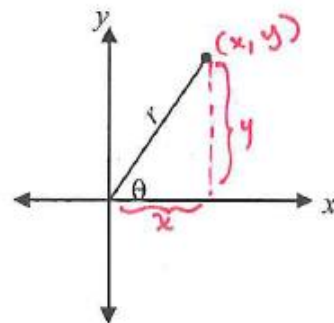
$$(x, y) \rightarrow (r, \theta)$$

$$x^2 + y^2 = r^2$$

* then, solve for r

$$\tan \theta = \frac{y}{x}$$

* then, solve for θ



Example 5:

Convert $(3\sqrt{2}, 3\sqrt{2})$ to polar coordinates.

$$x^2 + y^2 = r^2$$

$$(3\sqrt{2})^2 + (3\sqrt{2})^2 = r^2$$

$$18 + 18 = r^2$$

$$36 = r^2$$

$$6 = r$$

$$\tan \theta = \frac{3\sqrt{2}}{3\sqrt{2}}$$

$$\tan \theta = 1$$

$$\theta = \frac{\pi}{4}$$

$$\frac{\sin \theta}{\cos \theta} = 1$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

... 😊

$$\boxed{(6, \frac{\pi}{4})}$$

More Practice

Polar Coordinates

http://mathinsight.org/polar_coordinates

<https://www.mathsisfun.com/polar-cartesian-coordinates.html>

[http://math.illinois.edu/~rasekh2/math231\(s2016\)/PolarEquations.pdf](http://math.illinois.edu/~rasekh2/math231(s2016)/PolarEquations.pdf)

<http://tutorial.math.lamar.edu/Classes/CalcII/PolarCoordinates.aspx>

http://www.mathwords.com/p/polar_rectangular_conversion_formulas.htm

<https://youtu.be/r0fv9V9GHdo>

<https://youtu.be/jexMSISDubM>

<https://youtu.be/2RQk9P-EVpQ>

<https://youtu.be/L4v98ZZft68>

Homework Assignment

p.539 #1-7 odd,13,15,21,27,29