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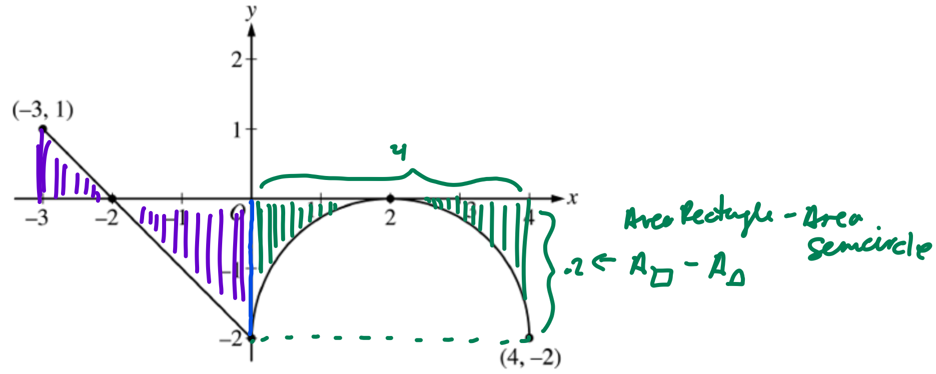
Properties of Definite Integrals



Graph of $f(x)$

Let f be the function, given by the graph above, defined on the closed interval $-3 \le x \le 4$ which consists of one line segment and a semicircle. Let $g(x) = \int_0^x f(t)\,dt$.

Find $g(0)$, $g(4)$, and $g(-3)$.

$g(0) = \int_0^0 f(t)\,dt$

$\boxed{g(0) = 0}$

$g(4) = \int_0^4 f(t)\,dt$

$= 4(-2) - -\frac{1}{2}\pi(2)^2$

$\boxed{g(4) = -8 + 4\pi}$

$g(-3) = \int_0^{-3} f(t)\,dt$

$= -\int_0^{-3} f(t)\,dt$

$= -\left(\frac{1}{2}(1)(2) + -\frac{1}{2}(2)(2)\right)$

$= -(1 - 2)$

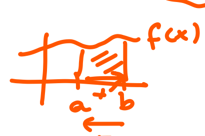
$= -(-1)$

$\boxed{g(-3) = 1}$

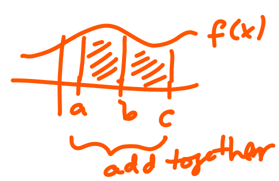
Properties of Definite Integrals

$\int_a^a f(x)\,dx = 0$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

variable coefficient variable

$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Example:

Given $\int_{-5}^{5} f(x)\,dx = 14$, $\int_{5}^{8} f(x)\,dx = -17$, and $\int_{-5}^{5} h(x)\,dx = 24$, find:

a) $\int_{5}^{-5} f(x)\,dx = -\int_{-5}^{5} f(x)\,dx$

$= -(14)$

$= \boxed{-14}$

b) $\int_{-5}^{5} (3f(x) - 2h(x))dx = \int_{-5}^{5} 3f(x)\,dx - \int_{-5}^{5} 2h(x)\,dx$

$= 3\int_{-5}^{5} f(x)\,dx - 2\int_{-5}^{5} h(x)\,dx$

$= 3(14) - 2(24)$

$= 42 - 48$

$= \boxed{-6}$

c) $\int_{-5}^{8} f(x)\,dx = \int_{-5}^{5} f(x)\,dx + \int_{5}^{8} f(x)\,dx$

$= 14 + (-17)$

$= \boxed{-3}$