DATE: ____________

Even though the study of differential equations is complex, we are only required to know and understand the easiest method for solving them – **separation of variables**.

**1.** What makes the equation $\frac{dy}{dx} = \frac{x}{y^2}$ a differential equation?

It's a derivative ($\frac{dy}{dx}$)

**2.** Your goal is going to be to find an equation (in the form $y = f(x)$) whose derivative is $\frac{x}{y^2}$. Why can't you just simply integrate right away? Think about what makes this differential equation different from $\frac{dy}{dx} = \frac{4}{x^2}$, which you can just simply integrate right away.

b/c you move dx, but then have $y^2$... can't antiderive w/ 2 different variables

$$dy = \frac{x}{y^2}dx$$

← can't integrate this

**3.** Before you can integrate $\frac{dy}{dx} = \frac{x}{y^2}$, you must separate the variables (put the $y$'s on one side and the $x$'s on the other). How can you accomplish this? Show your work in separating the variables.

$dy = \frac{x}{y^2}dx$ multiply by dx

$y^2 dy = x\,dx$ multiply by $y^2$

**4.** Once the variables are separated, you can integrate both sides. ~~And~~ Write the general solution to the differential equation.

$$\int y^2\,dy = \int x\,dx$$

$$\frac{1}{3}y^3 = \frac{1}{2}x^2 + C$$

$$y^3 = \frac{3}{2}x^2 + C$$

$$y = \sqrt[3]{\frac{3}{2}x^2 + C}$$

*Example* 1:

Let $y = f(x)$ be the particular solution to the given differential equation $\frac{dy}{dx} = 2xy + 2x$ with the initial condition $f(1) = 2$. Find $y = f(x)$.

$$\frac{dy}{dx} = 2xy + 2x$$

$$dy = (2xy + 2x)dx$$

$$dy = 2x(y+1)\,dx$$

$$\frac{1}{y+1}\,dy = 2x\,dx$$

$$\ln|y+1| = x^2 + C$$

general solution ← use $f(1)=2$ to get $C$.

$$\ln|2+1| = 1^2 + C$$

$$\ln 3 = 1 + C$$

$$-1 + \ln 3 = C$$

$$\ln|y+1| = x^2 - 1 + \ln 3$$

$$e^{\ln|y+1|} = e^{x^2 - 1 + \ln 3}$$

$$|y+1| = e^{x^2} \cdot e^{-1} \cdot e^{\ln 3}$$

$$|y+1| = 3e^{x^2-1}$$

$$y+1 = \pm 3e^{x^2-1}$$

$$y+1 = 3e^{x^2-1}$$

since $y+1>0$ where $y=2$: $2+1>0$, $3>0$, keep "+"

$$\boxed{y = 3e^{x^2-1} - 1}$$

*Example* 2:

Find the particular solution to the differential equation $\frac{dy}{dx} = e^{x-2y}$ that passes through the origin.

origin → $(0,0)$

$$dy = e^{x-2y}\,dx$$

$$dy = \frac{e^x}{e^{2y}}\,dx$$

$$\int e^{2y}\,dy = \int e^x\,dx$$

$$\frac{1}{2}e^{2y} = e^x + C$$

← use initial condition

$$\frac{1}{2}e^{2(0)} = e^0 + C$$

$$\frac{1}{2} = 1 + C$$

$$-\frac{1}{2} = C$$

$$\frac{1}{2}e^{2y} = e^x + C$$

$$\frac{1}{2}e^{2y} = e^x - \frac{1}{2}$$

$$e^{2y} = 2e^x - 1$$

$$\cancel{\ln e}^{2y} = \ln|2e^x - 1|$$

$$2y = \ln|2e^x - 1|$$

$$\boxed{y = \frac{1}{2}\ln|2e^x - 1|}$$