DATE: _____

1. Let f be a function with $f(1)=-4$ such that for all points (x,y) on the graph of f the slope is given by $\dfrac{3x^2+1}{2y}$. Find $f(x)$.

$\dfrac{dy}{dx} = \dfrac{3x^2+1}{2y}$

$\int 2y\,dy = \int (3x^2+1)\,dx$

$y^2 = x^3 + x + C \qquad (1,-4)$

$(-4)^2 = 1^3 + 1 + C$

$16 = 2 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \pm\sqrt{x^3+x+14}$ ← keep "−" b/c $y<0$ when $x=1$

$y = -\sqrt{x^3+x+14}$

2. The function f is differentiable for all real numbers. The point $\left(3,\dfrac{1}{4}\right)$ is on the graph of $y=f(x)$, and the slope at each point (x,y) on the graph is given by $\dfrac{dy}{dx}=y^2(6-2x)$. Find $y=f(x)$ by solving the differential equation $\dfrac{dy}{dx}=y^2(6-2x)$ with the initial condition $f(3)=\dfrac{1}{4}$

$\dfrac{dy}{dx} = y^2(6-2x)$

$\int \frac{1}{y^2}\,dy = \int (6-2x)\,dx$

$-y^{-1} = 6x - x^2 + C \qquad (3,\frac{1}{4})$

$-(\frac{1}{4})^{-1} = 6(3) - 3^2 + C$

$-4 = 18 - 9 + C$

$-4 = 9 + C$

$13 = C$

$-y^{-1} = 6x - x^2 + 13$

$-\dfrac{1}{y} = 6x - x^2 + 13$

$-y = \dfrac{1}{6x-x^2+13}$

$y = \dfrac{1}{x^2-6x-13}$