

## Exponential Growth &amp; Decay

1. If  $\frac{dy}{dx} = \frac{e^x}{y^2}$  and  $y(0) = 1$ , find an equation for  $y$  in terms of  $x$ .

$$dy = \frac{e^x}{y^2} dx$$

$$\int y^2 dy = \int e^x dx$$

$$\frac{1}{3} y^3 = e^x + C \rightarrow \frac{1}{3} y^3 = e^x - \frac{2}{3}$$

$$\frac{1}{3} (1)^3 = e^0 + C$$

$$y^3 = 3e^x - 2$$

$$\frac{1}{3} = 1 + C$$

$$-\frac{2}{3} = C$$

$$y = \sqrt[3]{3e^x - 2}$$

2. If  $\frac{dy}{dx} = \frac{\sin x}{\cos y}$  and  $y(0) = \frac{3\pi}{2}$ , find an equation for  $y$  in terms of  $x$ .

$$dy = \frac{\sin x}{\cos y} dx$$

$$\int \cos y dy = \int \sin x dx$$

$$\sin y = -\cos x + C \rightarrow \sin y = -\cos x + 0$$

$$\sin \frac{3\pi}{2} = -\cos 0 + C$$

$$\sin y = -\cos x$$

$$-1 = -1 + C$$

$$0 = C$$

$$y = \sin^{-1}(-\cos x)$$

3. A radioactive element decays exponentially proportionally to its mass. One-half of its original amount remains after 5,750 years. If 10,000 grams of the element are present initially, how much will be left after 1,000 years?

$$y = Ce^{kt}$$

$$y = 10000e^{kt}$$

$$5000 = 10000e^{k(5750)}$$

$$\frac{1}{2} = e^{5750k}$$

$$\ln\left(\frac{1}{2}\right) = 5750k$$

$$\frac{\ln\left(\frac{1}{2}\right)}{5750} = k$$

~~(0, 10000)~~

(0, 10000)

(5750, 5000)

(1000, ?)

$$y = 10000e^{\left(\frac{\ln \frac{1}{2}}{5750}\right)(1000)}$$

$$y = 8864.351 \text{ grams}$$

## AP CALCULUS AB – Practice Problem

A graphing calculator is required for some problems or parts of problems.

The number of gallons,  $P(t)$ , of a pollutant in a lake changes at the rate  $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$  gallons per day, where  $t$  is measured in days. There are 50 gallons of the pollutant in the lake at time  $t = 0$ . The lake is considered safe when it contains 40 gallons or less of pollutant.

- Is the amount of pollutant increasing at time  $t = 9$ ? Why or why not?
- For what value of  $t$  will the number of gallons of pollutant be at its minimum? Justify your answer.
- Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- An investigator uses the tangent line approximation to  $P(t)$  at  $t = 0$  as a model for the amount of pollutant in the lake. At what time  $t$  does this model predict that the lake becomes safe?

$$a) \quad P'(t) = 1 - 3e^{-.2\sqrt{t}}$$

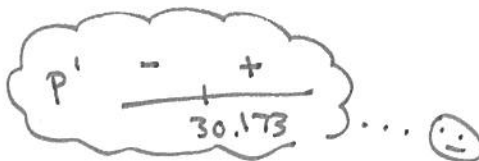
$$P'(9) = 1 - 3e^{-.2\sqrt{9}} = -.646 < 0$$

Since  $P'(9) < 0$ , the amount of pollutant is not increasing @  $t = 9$ .

$$b) \quad P'(t) = 0 \text{ (crit \#)}$$

$$1 - 3e^{-.2\sqrt{t}} = 0$$

$$t = 30.173$$



# of gallons of pollutant is @ minimum  
 @  $t = 30.173$  days b/c  $P'(t)$  changes  
 from neg. to pos. @  $t = 30.173$  days

c) lake safe when 40 gallons or less of pollutant

$$\begin{aligned} \text{when } t=30.173 \quad \text{pollutant} &= \underbrace{50}_{\text{initial gallons}} + \int_0^{30.173} P'(t) dt \\ &= 50 + (-14.896) \\ &= 35.104 \end{aligned}$$

Since 35.104 gallons < 40 gallons, the lake is safe when the # of pollutants is a min.

d)  $y - y_1 = m(x - x_1)$

$$P(t) - 50 = P'(0)(t - 0)$$

$$P(t) - 50 = -2t$$

$$P(t) = -2t + 50$$

$$-2t + 50 \leq 40$$

$$-2t \leq -10$$

$$t \geq 5$$

when  $t=5$ , tangent line approximation predicts the lake is safe.

$$\begin{aligned} P'(0) &= 1 - 3e^{-2\sqrt{0}} \\ &= 1 - 3 \\ &= -2 \end{aligned}$$