DATE: __________

Exponential Growth & Decay

1. If $\dfrac{dy}{dx} = \dfrac{e^x}{y^2}$ and $y(0) = 1$, find an equation for y in terms of x.

$dy = \dfrac{e^x}{y^2}\,dx$

$\int y^2\,dy = \int e^x\,dx$

$\frac{1}{3}y^3 = e^x + C \longrightarrow \frac{1}{3}y^3 = e^x - \frac{2}{3}$

$\frac{1}{3}(1)^3 = e^0 + C$

$y^3 = 3e^x - 2$

$\frac{1}{3} = 1 + C$

$\boxed{y = \sqrt[3]{3e^x - 2}}$

$-\frac{2}{3} = C$

2. If $\dfrac{dy}{dx} = \dfrac{\sin x}{\cos y}$ and $y(0) = \dfrac{3\pi}{2}$, find an equation for y in terms of x.

$dy = \dfrac{\sin x}{\cos y}\,dx$

$\int \cos y\,dy = \int \sin x\,dx$

$\sin y = -\cos x + C \longrightarrow \sin y = -\cos x + 0$

$\sin \frac{3\pi}{2} = -\cos 0 + C$

$\sin y = -\cos x$

$-1 = -1 + C$

$\boxed{y = \sin^{-1}(-\cos x)}$

$0 = C$

3. A radioactive element decays exponentially proportionally to its mass. One-half of its original amount remains after 5,750 years. If 10,000 grams of the element are present initially, how much will be left after 1,000 years?

$y = Ce^{kt}$

$(0, 10000)$

$y = 10000e^{kt}$

$(5750, 5000)$

$5000 = 10000e^{k(5750)}$

$(1000, ?)$

$\frac{1}{2} = e^{5750k}$

$y = 10000e^{\left(\frac{\ln\frac{1}{2}}{5750}\right)(1000)}$

$\ln\left(\frac{1}{2}\right) = 5750k$

$\boxed{y = 8864.351 \text{ grams}}$

$\dfrac{\ln\left(\frac{1}{2}\right)}{5750} = k$

AP CALCULUS AB – Practice Problem

A graphing calculator is required for some problems or parts of problems.

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered safe when it contains 40 gallons or less of pollutant.

a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake At what time t does this model predict that the lake becomes safe?

a) $P'(t) = 1 - 3e^{-.2\sqrt{t}}$

$P'(9) = 1 - 3e^{-.2\sqrt{9}} = -.646 < 0$

Since $P'(9) < 0$, the amount of pollutant is not increasing @ $t = 9$.

b) $P'(t) = 0$ (crit #)

$1 - 3e^{-.2\sqrt{t}} = 0$

$t = 30.173$

P' $-$ $+$ 30.173 ...

# of gallons of pollutatant is @ minimum @ $t = 30.173$ days b/c $P'(t)$ changes from neg. to pos. @ $t = 30.173$ days

c) lake safe when 40 gallons or less of pollutant

when $t = 30.173$ $\quad$ pollutant $= \underbrace{50}_{\text{initial gallons}} + \int_0^{30.173} P'(t)\,dt$

$= 50 + (-14.896)$

$= 35.104$

Since 35.104 gallons $<$ 40 gallons, the lake is safe when the # of pollutants is a min.

d) $y - y_1 = m(x - x_1)$

$P(t) - 50 = P'(0)(t - 0)$ $\qquad$ $P'(0) = 1 - 3e^{-.2\sqrt{0}}$

$= 1 - 3$

$= -2$

$P(t) - 50 = -2t$

$P(t) = -2t + 50$

$-2t + 50 \leq 40$

$-2t \leq -10$

$t \geq 5$ $\quad$ when $t = 5$, tangent line approximation predicts the lake is safe.