DATE:____________

1. Write the differential equation that models the following statement:
   The rate of change of Q with respect to t is inversely proportional to the square of t.

division

$$\frac{dQ}{dt} = \frac{k}{t^2}$$

2. The rate of change of V is proportional to V. When $t = 0$, V = 20,000 and when $t = 4$, V = 12,500. What is the value of V when $t = 6$?

$$\frac{dV}{dt} = kV$$

(0, 20000)
(4, 12500)
(6, ?)

$$dV = kV\,dt$$

$$\int \frac{1}{V}\,dV = \int k\,dt$$

$$\ln|V| = kt + C$$

$$\ln 20000 = k(0) + C$$

$$\ln 20000 = C$$

$$\ln|V| = kt + \ln 20{,}000$$

$$\ln 12500 = k(4) + \ln 20000$$

$$\ln 12500 - \ln 20000 = 4k$$

$$\frac{1}{4}\ln\left(\frac{12500}{20000}\right) = k$$

$$\ln|V| = \frac{1}{4}\ln\left(\frac{12000}{20000}\right)t + \ln 20000$$

$$e^{\ln V} = e^{\left[\frac{1}{4}\ln\left(\frac{12000}{20000}\right)\right](6) + \ln 20000}$$

$$\boxed{V = 9882.118}$$

3. The rate of change of the number of coyotes $N(t)$ in a population is directly proportional to 650 - $N(t)$, where t is the time in years. When $t = 0$, the population is 300, and when $t = 2$, the population has increased to 500. Find the population when $t = 3$.

$$\frac{dN}{dt} = k(650 - N)$$

(0, 300)
(2, 500)
(3, ?)

$$dN = k(650 - N)\,dt$$

$$\int \frac{1}{650 - N}\,dN = \int k\,dt$$

$$-\ln|650 - N| = kt + C$$

$$-\ln|650 - 300| = k(0) + C$$

$$-\ln(350) = C$$

$$-\ln|650 - N| = kt - \ln 350$$

$$-\ln|650 - 500| = k(2) - \ln 350$$

$$\ln 350 - \ln 150 = 2k$$

$$\frac{1}{2}\ln\left(\frac{350}{150}\right) = k \rightarrow k = \frac{1}{2}\ln\left(\frac{7}{3}\right)$$

$$-\ln|650 - N| = \frac{1}{2}\ln\frac{7}{3}\,t - \ln 350$$

$$-\ln|650 - N| = \left(\frac{1}{2}\ln\frac{7}{3}\right)(3) - \ln 350$$

$$e^{\ln|650 - N|} = e^{-\frac{3}{2}\ln\frac{7}{3} + \ln 350}$$

$$|650 - N| = e^{-\frac{3}{2}\ln 7/3 + \ln 350}$$

$$N = 551.802$$

$$\boxed{551 \text{ coyotes}}$$

4. A calf that weighs 60 pounds at birth gains weight at the rate $\frac{dw}{dt} = 1200 - w$ where w is weight in pounds and t is time in years. If the animal is sold when its weight reaches 800 pounds, find the time of sale.

(t, w)
$(0, 60)$
$(?, 800)$

$$\frac{dw}{dt} = 1200 - w$$
$$dw = (1200 - w)\,dt$$
$$\int \frac{1}{1200-w}\,dw = \int dt$$
$$-\ln|1200 - w| = t + C$$
$$-\ln|1200 - 60| = 0 + C$$
$$-\ln 1140 = C$$
$$-\ln|1200 - w| = t - \ln 1140$$
$$-\ln|1200 - 800| = t - \ln 1140$$
$$-\ln 400 = t - \ln 1140$$

$$\ln 1140 - \ln 400 = t$$
$$\ln\left(\frac{1140}{400}\right) = t$$
$$\boxed{1.047 \text{ years} = t}$$

5. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F), is taken out of an oven and placed in a 75°F room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F **(B)** 119°F **(C)** 147°F **(D)** 238°F **(E)** 335°F

$$\frac{dP}{dt} = -110e^{-.4t}$$

$P(0) = 350$
$P(5) = ?$

$$\int dP = \int -110e^{-.4t}\,dt$$
$$P(t) = \int -110e^{-.4t}\,dt$$
$$P(t) = \underbrace{350}_{\text{initial temp}} + \underbrace{\int_0^t -110e^{-.4t}\,dt}_{\text{change in temp over } t \text{ minutes}}$$
$$P(5) = 350 + \int_0^5 -110e^{-.4t}\,dt$$
$$P(5) = 112.217°\text{ F}$$