

## Polar Graphing Activity

You will explore graphs of polar equations. A polar equation is a function rule in the form  $r = f(\theta)$ , where  $\theta$  can be measured in radians or degrees.

What is the *independent* variable?

What is the *dependent* variable?

Use your calculator to explore the following:

1. Consider equations of the form:  $r = a \sin \theta$   
 $r = a \cos \theta$  Experiment with different values for  $a$ .

a. What type of figure is created by these equations? *circles*

b. How do the figures differ when different trig functions are used (sin vs. cos)?

*$r = a \sin \theta \rightarrow$  symmetrical about  $\pi/2$  axis (y-axis)       $r = a \cos \theta \rightarrow$  symmetrical about polar axis (x-axis)*

c. What is significant about the  $a$ -value?

*the  $a$ -value is the diameter.*

2. Consider equations of the form:  $r = a \pm b \sin \theta$   
 $r = a \pm b \cos \theta$  *Limaçons*

Graph together:  $r = 2 + 5 \sin \theta$   
 $r = 1 + 3 \cos \theta$

Graph together:  $r = 4 + 3 \sin \theta$   
 $r = 3 + 2 \cos \theta$

Graph together:  $r = 4 + 4 \sin \theta$   
 $r = 2 - 2 \cos \theta$

a. How do the figures differ when different trig functions are used (sin vs. cos)?

*when sin used, symmetrical about  $\pi/2$  axis  
 when cosine used, symmetrical about polar axis.*

b. What is it about the " $a$ " & " $b$ " values that determines the shape of the graph?

*when  $a < b$ , the graph has an inner loop.      when  $a = b$ , the graph is*

c. What is the significance of " $a + b$ "?

*" $a + b$ " is the largest value of radius ( $r$ )*

*a "cardioid" (heart shaped)*

3. Consider equations of the form:  $r = a \sin(n\theta)$   
 $r = a \cos(n\theta)$  *Rose Curves*

Graph these functions one at a time:  $r = 2 \sin(3\theta)$     $r = 4 \sin(2\theta)$     $r = 2 \cos(3\theta)$     $r = 4 \cos(2\theta)$

a. How do the figures differ when different trig functions are used (sin vs. cos)?

*when sin used, symmetrical about  $\pi/2$  axis  
 when cosine used, symmetrical about polar axis*

b. What determines the length of a petal?

*$a$  determines the length of a petal*

c. What determines the number of petals?

*if  $n$  is odd,  $n$  # of petals*

*if  $n$  is even,  $2n$  # of petals*

4. Consider equations of the form:

$$r = a\theta + b$$

$$r = ab^\theta$$

To see these graphs better, do the following: ZOOM 6; change  $\theta_{\max}$  to  $6\pi$ ; then ZOOM 5.

a. Graph these two:  $r = \theta + 2$       Then graph these two:  $r = 3^\theta$

$$r = 2\theta$$

$$r = 2 \cdot 3^\theta$$

*Spirals of Archimedes*

*Logarithmic Spirals*

c. What is the difference between the Spirals of Archimedes and Logarithmic Spirals?

*Spirals of Archimedes increase by factors of # or by adding #s*

*Logarithmic Spirals increase exponentially*

5. Graph each of the following, one at a time.

$$r = \frac{5}{6 + 8 \cos \theta} \quad r = \frac{6}{4 + 3 \cos \theta} \quad r = \frac{2}{3 + 3 \cos \theta}$$

a. What is the name of the shape for each figure produced?

*hyperbola, ellipse, parabola*

b. How are these equations related to those of the limaçons?

*Reciprocals of the limaçons*

## Polar Graphing Practice

1. What shape is the following graph?  $r = 8 \sin \theta$  Identify the center and radius. Then convert the equation into rectangular form.

circle

center:  $(0, 4)$

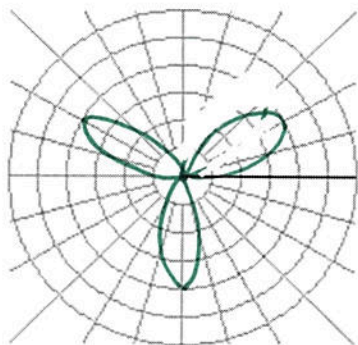
radius: 4

$$(x-0)^2 + (y-4)^2 = 4^2$$

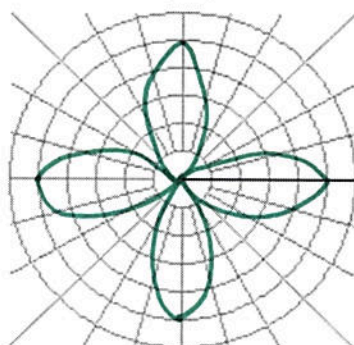
$$x^2 + (y-4)^2 = 16$$

2. Sketch accurate graphs of the following:

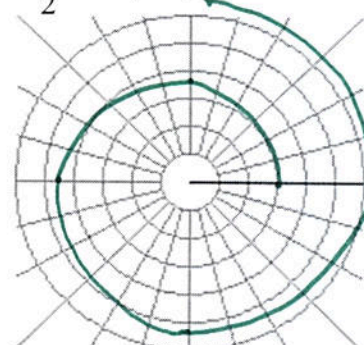
a.  $r = 4 \sin(3\theta)$



b.  $r = 5 \cos(2\theta)$



c.  $r = \frac{\theta}{2} + 3$  (Graph 2 revolutions.)



Write polar equations for the following:

3. A circle with radius 4.8 and oriented to the polar axis  $r = 9.6 \cos \theta$

4. An example of a logarithmic spiral  $r = 5^\theta$

5. A rose curve with 20 petals of length 13 units, oriented to the  $\pi/2$  axis  $r = 13 \sin(10\theta)$

use,  $2n = 20$   
 $n = 10$

$a = 13$

use  
Sine