

*AP Practice Problems*

1. Let  $g$  be a function with  $g(4) = 1$  such that all points  $(x, y)$  on the graph of  $g$  satisfy the logistic differential equation  $\frac{dy}{dx} = 2y(3 - y)$ .
- a) Find  $\lim_{x \rightarrow \infty} g(x)$  and  $\lim_{x \rightarrow \infty} g'(x)$ . (It is not necessary to solve for  $g(x)$  or show how you arrived at your answers.)
- b) For what value of  $y$  does the graph of  $g$  have a point of inflection? Find the slope of the graph of  $g$  at the point of inflection. (It is not necessary to solve for  $g(x)$ .)

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2. A population is modeled by the function  $P$  that satisfies the logistic differential equation:
- $$\frac{dP}{dt} = \frac{P}{5} \left( 1 - \frac{P}{12} \right).$$

a) If  $P(0) = 3$ , what is  $\lim_{t \rightarrow \infty} P(t)$ ? If  $P(0) = 20$ , what is  $\lim_{t \rightarrow \infty} P(t)$ ?

b) If  $P(0) = 3$ , for what value of  $P$  is the population growing the fastest?