AP Practice Problems

1. Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation $\frac{dy}{dx} = 2y(3 - y)$.

→ carrying capacity

a) Find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$. (It is not necessary to solve for $g(x)$ or show how you arrived at your answers.)

$\lim_{x\to\infty} g(x)$ is carrying capacity of $g(x)$, so $\boxed{\lim_{x\to\infty} g(x) = 3}$

$\frac{dy}{dx} = g'(x)$ and $g(x)$ is logistic equation, so as $x \to \infty$, $g'(x) \to 0$

$\therefore \boxed{\lim_{x\to\infty} g'(x) = 0}$

b) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)

→ dy/dx

g'' changes signs

$\frac{dy}{dx} = 6y - 2y^2$

$\frac{d^2y}{dx^2} = 6\frac{dy}{dx} - 4y\frac{dy}{dx}$

$0 = \frac{dy}{dx}(6 - 4y)$

$\frac{dy}{dx} = 0$ never happens

$6 - 4y = 0$

$6 = 4y$

$\frac{3}{2} = y$

$\frac{d^2y}{dx^2}$: + | − (① $\frac{3}{2}$ ②)

g has inf pt @ $y = 3/2$ b/c g'' changes signs @ $y = 3/2$

$\left.\frac{dy}{dx}\right|_{y=3/2} = 2(3/2)(3 - 3/2) = \boxed{\frac{9}{2}}$

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2. A population is modeled by the function P that satisfies the logistic differential equation: $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right)$.

→ carrying capacity

a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$? If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 3$, $\lim_{t\to\infty} P(t) = 12$

If $P(0) = 20$, $\lim_{t\to\infty} P(t) = 12$

b/c 12 is the carrying capacity

b) If $P(0) = 3$, for what value of P is the population growing the fastest?

→ inf pt

Population grows the fastest @ $\frac{1}{2}$ carrying capacity, or @ inf pt, which is when $P = 6$.