DATE: _____

Logistic Differential Equations

Suppose that a number of rabbits are introduced onto an island on which they have no natural enemies but that can support a maximum population of 1000 rabbits. Let $P(t)$ denote the number of rabbits at time *t* (measured in months) and suppose that *P* satisfies the differential equation $\frac{dP}{dt} = kP(1000 - P)$, where *k* is a positive constant.

a) Suppose that 1000 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

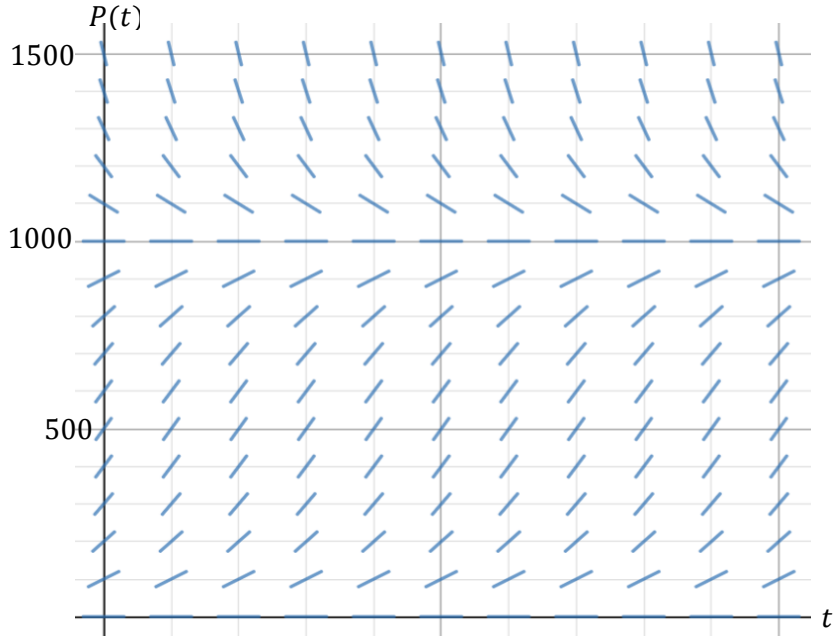
b) Suppose that 1500 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

c) Suppose that 250 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

d) Find the value of *P* for which the rate of change of the rabbit population is maximal. (HINT: Evaluate $\frac{d}{dP}(P')$)

So, What do those answers mean?

Suppose that a number of rabbits are introduced onto an island on which they have no natural enemies but that can support a maximum population of 1000 rabbits. Let $P(t)$ denote the number of rabbits at time *t* (measured in months) and suppose that *P* satisfies the differential equation $\frac{dP}{dt} = kP(1000 - P)$, where *k* is a positive constant.



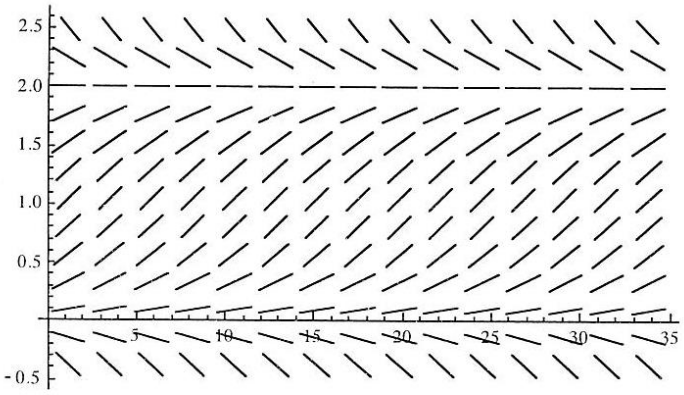
Logistic Differential Equation

$$\frac{dP}{dt} = kP(M - P)$$

OR

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

Example 1:

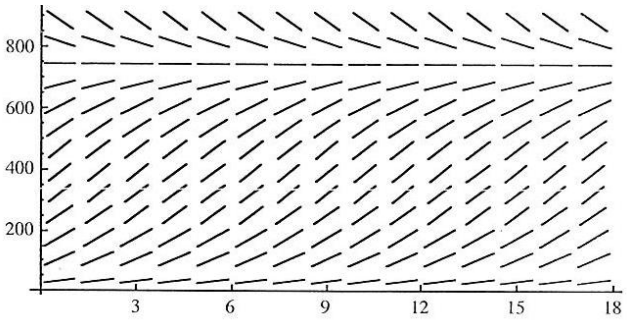


The differential equation $\frac{dP}{dt} = 0.15P(2 - P)$ has the slope field shown above. Sketch two possible solution curves, one with $P(0) = 0.1$ and one with $P(0) = 0.5$.

a) What is the carrying capacity of the differential equation?

b) For what population does the maximum rate of growth occur?

Example 2:



The differential equation $\frac{dF}{dt} = 4.5F(1 - \frac{F}{750})$ has the slope field shown above. Sketch two possible solution curves, one with $F(0) = 100$ and one with $F(0) = 200$.

a) What is the carrying capacity of the differential equation?

b) For what population does the maximum rate of growth occur?