DATE: ____________

## Logistic Differential Equations

Suppose that a number of rabbits are introduced onto an island on which they have no natural enemies but that can support a maximum population of 1000 rabbits. Let $P(t)$ denote the number of rabbits at time *t* (measured in months) and suppose that *P* satisfies the differential equation $\boxed{\frac{dP}{dt} = kP(1000 - P)}$ where *k* is a positive constant.

**a)** Suppose that 1000 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

↳ $\frac{dP}{dt} > 0$? $< 0$? $= 0$?

$\left.\frac{dP}{dt}\right|_{t=0} = k(1000)(1000 - 1000) = 0$

The rabbit population will remain constant b/c $\frac{dP}{dt} = 0$ when $t = 0, P = 1000$.

**b)** Suppose that 1500 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

$\left.\frac{dP}{dt}\right|_{t=0} = k(1500)(1000 - 1500) < 0$

The rabbit population will decrease b/c $\frac{dP}{dt} < 0$ when $t = 0, P = 1500$.

**c)** Suppose that 250 rabbits are introduced onto the island at *t* = 0. Does the model predict that the rabbit population will increase, decrease, or remain constant? Justify your answer.

$\left.\frac{dP}{dt}\right|_{t=0} = k(250)(1000 - 250) > 0$

The rabbit population will increase b/c $\frac{dP}{dt} > 0$ when $t = 0, P = 250$

**d)** Find the value of *P* for which the rate of change of the rabbit population is maximal. (HINT: Evaluate $\frac{d}{dP}(P')$ )

↳ 2nd derivative test

$\frac{dP}{dt} = 1000kP - kP^2$

$\frac{d}{dP}\left(\frac{dP}{dt}\right) = 1000k - 2Pk$ → $\frac{d}{dP}\left(\frac{d}{dP}\left(\frac{dP}{dt}\right)\right) = -2k$

$0 = 1000k - 2Pk$

$2Pk = 1000k$

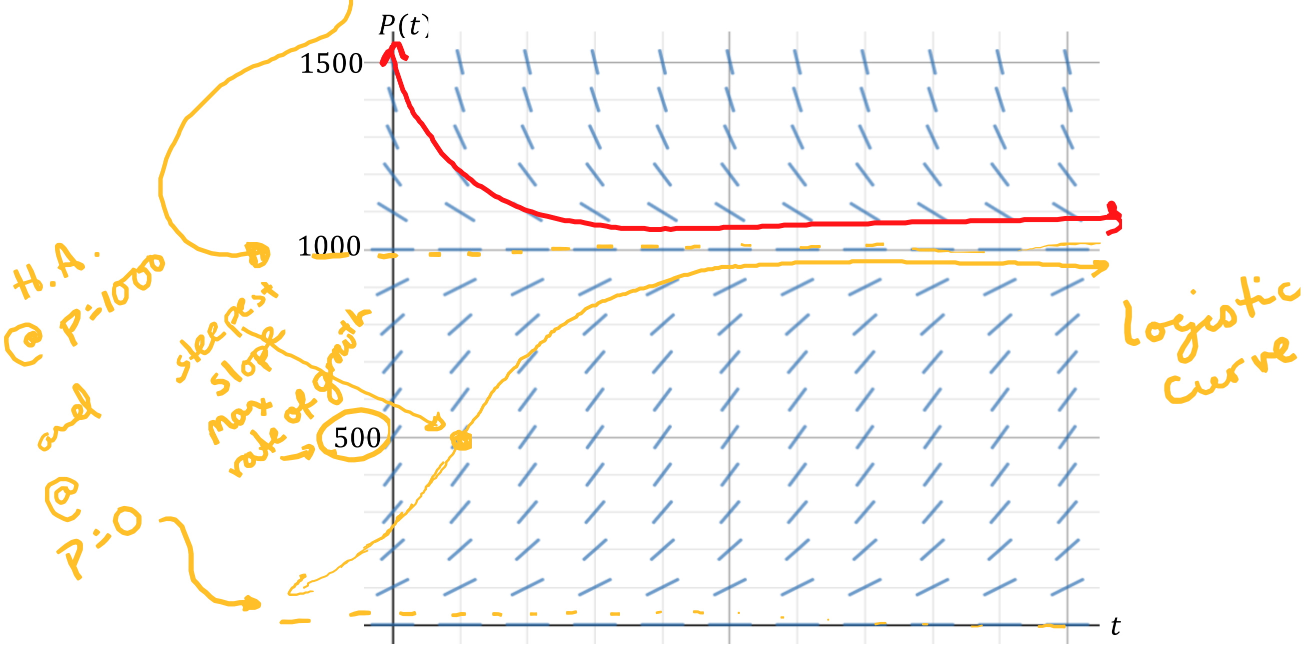
$P = 500$

$\left.\frac{d^2}{dP^2}\left(\frac{dP}{dt}\right)\right|_{P=500} = -2k < 0$

$\therefore$, $P = 500$ is a max

So, What do those answers mean?

Suppose that a number of rabbits are introduced onto an island on which they have no natural enemies but that can support a maximum population of 1000 rabbits. Let $P(t)$ denote the number of rabbits at time $t$ (measured in months) and suppose that $P$ satisfies the differential equation $\frac{dP}{dt} = kP(1000 - P)$, where $k$ is a positive constant.




## Logistic Differential Equation

$$\frac{dP}{dt} = kP(M - P)$$

P = population
M = carrying capacity
k = growth constant
t = time

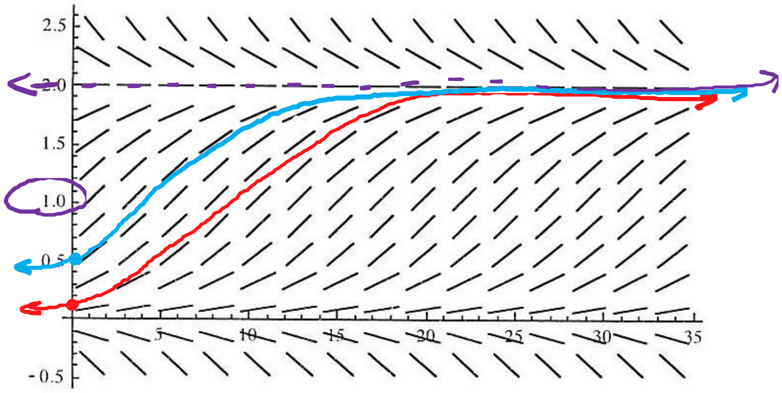
OR

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

$\frac{dP}{dt} = kP(M-P)$
$= kPM(1-\frac{P}{M})$
$= kP(1-\frac{P}{M})$

*Example 1:*

H.A. @ $P = 2.0$




$\frac{dP}{dt} = kP(M-P)$

The differential equation $\frac{dP}{dt} = 0.15P(2 - P)$ has the slope field shown above. Sketch two possible solution curves, one with $P(0) = 0.1$ and one with $P(0) = 0.5$.

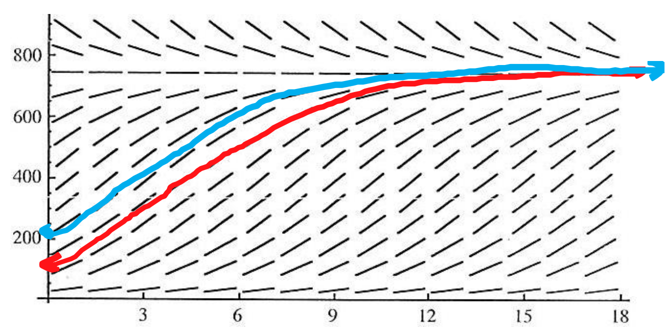
**a)** What is the carrying capacity of the differential equation?

Carrying capacity is 2.0

**b)** For what population does the maximum rate of growth occur? $\frac{1}{2}$ carrying capacity

Max rate of growth occurs @ inf pt when population = 1.0

*Example 2:*




$\frac{dF}{dt} = kF(M-F)$
$= kFM(1 - \frac{F}{M})$
$= kF(1 - \frac{F}{A})$

The differential equation $\boxed{\frac{dF}{dt} = 4.5F(1 - \frac{F}{750})}$ has the slope field shown above. Sketch two possible solution curves, one with $F(0) = 100$ and one with $F(0) = 200$.

**a)** What is the carrying capacity of the differential equation?

Carrying capacity is 750.

**b)** For what population does the maximum rate of growth occur? $\frac{1}{2}$ carrying capacity

Max rate of growth occurs @ inf pt when pop is 375.