Date: ___________

6.5 Logistic Growth Practice

1. Consider the function $f(x) = \frac{1}{x^2 - kx}$, where k is a nonzero constant. Let $k = 6$, find $\int f(x)\,dx$.

$\int \frac{1}{x^2-6x}\,dx$

$\frac{1}{x(x-6)} = \frac{A}{x} + \frac{B}{x-6}$

$1 = Ax - 6A + Bx$

$A + B = 0 \qquad 1 = -6A$

$B = 1/6 \qquad -\frac{1}{6} = A$

$= \int \left(\frac{-1/6}{x} + \frac{1/6}{x-6}\right) dx$

$= -\frac{1}{6}\int \frac{1}{x}\,dx + \frac{1}{6}\int \frac{1}{x-6}\,dx$

$= -\frac{1}{6}\ln|x| + \frac{1}{6}\ln|x-6| + c$

$= \frac{1}{6}(-\ln|x| + \ln|x-6|) + c$

$= \boxed{\frac{1}{6}\ln\left|\frac{x-6}{x}\right| + c}$ or $\ln \sqrt[6]{\left|\frac{x-6}{x}\right|} + c$

2. Find the area between the curve $f(x) = \frac{25}{x^2+3x-4}$ and the x-axis from $x = 3$ to $x = 5$.

$\int_3^5 \frac{25}{x^2+3x-4}\,dx$

$\frac{25}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$

$25 = Ax - A + Bx + 4B$

$A + B = 0 \qquad A = -5$

$-A + 4B = 25$

$5B = 25$

$B = 5$

$= \int_3^5 \left(\frac{-5}{x+4} + \frac{5}{x-1}\right) dx$

$= \left(-5\ln|x+4| + 5\ln|x-1|\right)\Big|_3^5$

$= 5\left(\ln\left|\frac{x-1}{x-4}\right|\right)\Big|_3^5$

$= 5(\ln 4 - \ln 2)$

$= \boxed{5\ln 2}$

3. The velocity function of a particle is described by $v(t) = \frac{\cos t}{\sin^2 t - \sin t}$. Find the position function of the particle.

$\int v(t)\,dt = \int \frac{\cos t}{\sin^2 t - \sin t}\,dt$

$x(t) = \int \frac{\cos t}{\sin t(\sin t - 1)}\,dt$

$u = \sin t$

$du = \cos t\,dt$

$= \int \frac{1}{u(u-1)}\,du$

$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$1 = Au - A + Bu$

$A + B = 0 \qquad 1 = -A$

$B = 1 \qquad -1 = A$

$= \int \left(\frac{-1}{u} + \frac{1}{u-1}\right) du$

$= -\ln|u| + \ln|u-1| + C$

$= \ln\left|\frac{u-1}{u}\right| + C$

$\boxed{x(t) = \ln\left|\frac{\sin t - 1}{\sin t}\right| + C}$