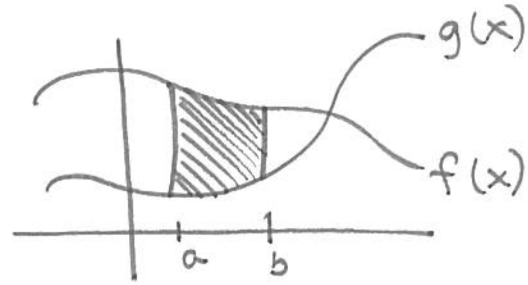
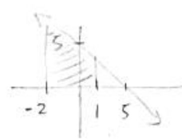
Area between Curves

$$\text{Area} = \int_a^b (\underbrace{f(x)}_{\substack{\uparrow \\ \text{top} \\ \text{curve}}} - \underbrace{g(x)}_{\substack{\uparrow \\ \text{bottom} \\ \text{curve}}})\,dx$$



1. Find the area of the region under each curve and bounded by $x = 1$ and $x = -2$.

a) $y = -x + 5$



$$\text{Area} = \int_{-2}^{1} (-x+5)\,dx$$

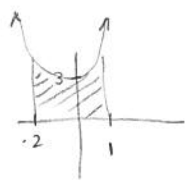
$$= \left(-\tfrac{1}{2}x^2 + 5x\right)\Big|_{-2}^{1}$$

$$= -\tfrac{1}{2}(1)^2 + 5(1) - \left(-\tfrac{1}{2}(-2)^2 + 5(-2)\right)$$

$$= -\tfrac{1}{2} + 5 + 2 + 10$$

$$= \boxed{16.5}$$

b) $y = x^2 + 3$



$$\text{Area} = \int_{-2}^{1} (x^2+3)\,dx$$

$$= \left(\tfrac{1}{3}x^3 + 3x\right)\Big|_{-2}^{1}$$

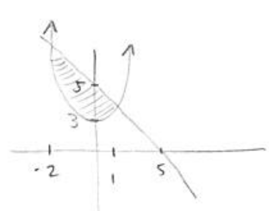
$$= \tfrac{1}{3}(1)^3 + 3(1) - \left(\tfrac{1}{3}(-2)^3 + 3(-2)\right)$$

$$= \tfrac{1}{3} + 3 + \tfrac{8}{3} + 6$$

$$= \boxed{12}$$

2. Find the area between $y = -x + 5$ and $y = x^2 + 3$ from $x = 1$ to $x = -2$.

$$\text{Area} = \int_{-2}^{1} (-x+5)\,dx - \int_{-2}^{1} (x^2+3)\,dx$$



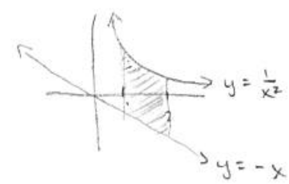
$$= 16.5 - 12$$

$$= \boxed{4.5}$$

Area Between Curves

Examples

1. Find the area of the region bounded by the graphs of $y = \frac{1}{x^2}$, $y = -x$, $x = 1$, and $x = 2$.



$$\text{Area} = \int_1^2 \left(\frac{1}{x^2} - (-x) \right) dx$$

$$= \int_1^2 (x^{-2} + x)\, dx$$

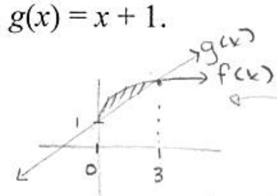
$$= \left(-x^{-1} + \frac{1}{2}x^2\right)\Big|_1^2$$

$$= \left(-2^{-1} + \frac{1}{2}(2)^2\right) - \left(-1^{-1} + \frac{1}{2}(1)^2\right)$$

$$= -\frac{1}{2} + 2 + 1 - \frac{1}{2}$$

$$= 3 - 1 = \boxed{2}$$

2. Find the area of the region bounded by the graphs of $f(x) = \sqrt{3x} + 1$, $g(x) = x + 1$.



$$\text{Area} = \int_0^3 \left(\sqrt{3x} + 1 - (x+1)\right) dx$$

$$= \int_0^3 (\sqrt{3x} + 1 - x - 1)\, dx$$

$$= \int_0^3 (\sqrt{3x} - x)\, dx$$

$$= \int_0^3 (\sqrt{3}\sqrt{x} - x)\, dx$$

$$= \int_0^3 (\sqrt{3}\, x^{1/2} - x)\, dx$$

$$= \left(\sqrt{3}\left(\tfrac{2}{3}x^{3/2}\right) - \tfrac{1}{2}x^2\right)\Big|_0^3$$

$$= \frac{2\sqrt{3}}{3}(3)^{3/2} - \frac{1}{2}(3)^2 - (0)$$

$$= \frac{2\sqrt{3}}{3} \cdot \sqrt{3}^{\,3} - \frac{9}{2} = \boxed{1.5}$$

need intersection pts!

set $f(x) = g(x)$

$\sqrt{3x} + 1 = x + 1$

$\sqrt{3x} = x$

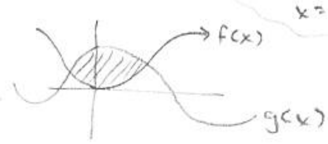
$3x = x^2$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0,\ x = 3$

3. Find the area of one of the regions bounded by $f(x) = \sin x$ and $g(x) = \cos x$.



$$\text{Area} = \int_{-\pi/4}^{\pi/4} (\cos x - \sin x)\, dx$$

$$= (\sin x + \cos x)\Big|_{-\pi/4}^{\pi/4}$$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \left(\sin(-\pi/4) + \cos(-\pi/4)\right)$$

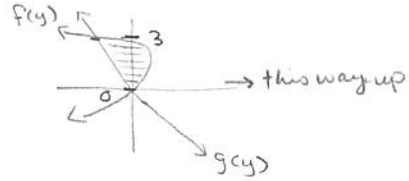
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\sqrt{2}}$$

$\cos x = \sin x$

when $x = \pi/4, -\pi/4$ or $x = \pi/4 + \frac{5\pi}{4}$

4. Find the area of the region bounded by $f(y) = 2y - y^2$ and $g(y) = -y$.



$$\text{Area} = \int_0^3 \left(2y - y^2 - (-y)\right) dy$$

$$= \int_0^3 (2y - y^2 + y)\, dy$$

$\int^3$