

7.2 Matrix Algebra

Target 8E: Represent a system of linear equations as a single matrix equation in a vector variable
 Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Review of Prior Concepts

Find the value of the variables.

$$1. \begin{bmatrix} 3 \\ 2k \end{bmatrix} + \begin{bmatrix} m \\ -4 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix} \quad 2. \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} \quad 3. \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}$$

More Practice

Solving Equal Matrices

<http://www.purplemath.com/modules/matrices3.htm>

<http://www.onlinemathlearning.com/equal-matrices.html>

<http://math.tutorvista.com/algebra/equal-matrices.html>

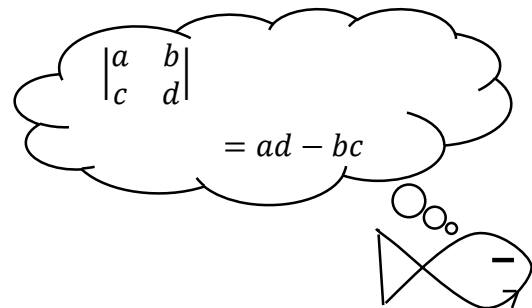
<https://www.youtube.com/watch?v=nOIRFQsHGOg>

<https://www.youtube.com/watch?v=LBfv7LtZHWk>

Determinant of a 2×2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of A is:

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ = ad - bc$$



Examples:

1. Find $\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}$

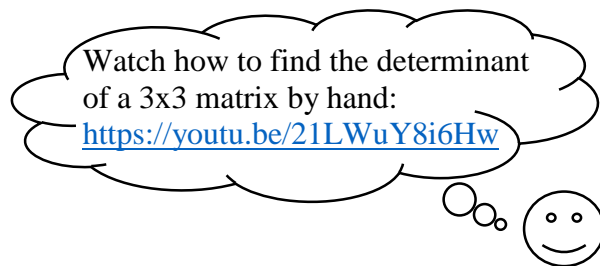
2. Find $\det A$, where $A = \begin{bmatrix} 6 & 3 \\ 1 & -2 \end{bmatrix}$

3. Using your graphing calculator, find $\begin{vmatrix} 3 & -1 & 7 \\ 2 & 0 & 1 \\ 1 & 4 & 2 \end{vmatrix}$.

menu

Watch how to find the determinant of a 3×3 matrix by hand:

<https://youtu.be/21LWuY8i6Hw>



Identity Matrix (for a square matrix)

$$A \cdot I_n = I_n \cdot A = A$$

$$\text{where } I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

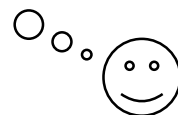
Examples:

1. Write I_2

2. Write I_4

Additive Identity: $x + 0 = x$ and $0 + x = x$

Multiplicative Identity: $x \cdot 1 = x$ and $1 \cdot x = x$



Inverse of a Square Matrix

If A and B are both $n \times n$ matrices and $BA = I_n$,
then B is the inverse of A (B is A^{-1}).

$$AB = I_n$$

$$BA = I_n$$

$$A() = I_n$$

$$()A = I_n$$

★ Matrix A has an inverse if $\det A \neq 0$

Find the Inverse of a 2×2 Matrix

If $\det A \neq 0$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$\begin{aligned} A^{-1} &= \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \end{aligned}$$

Examples:

1. Find the inverse of $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ and check your answer.

Check: $AB = BA = I_n$?

2. $A = \begin{bmatrix} 6 & 3 \\ 10 & 5 \end{bmatrix}$. Find A^{-1} .

3. Find inverse of: $\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

 or 

Watch how to find the inverse of a 3x3 matrix by hand:

https://youtu.be/YvjkPF6C_LI



More Practice

Inverse Matrices

<https://www.mathsisfun.com/algebra/matrix-inverse.html>

https://www.khanacademy.org/math/precalculus/precalc-matrices/practice-finding-inverses-of-2x2-matrices/e/matrix_inverse_2x2

<http://www.intmath.com/matrices-determinants/5-inverse-matrix.php>

https://youtu.be/OU9sWHk_dlw

https://youtu.be/y4B_EC5MNS8

Determinants

<http://www.mathsisfun.com/algebra/matrix-determinant.html>

<http://www.virtualnerd.com/algebra-2/matrices/determinants-cramers-rule/determinants/determinant-2-by-2>

https://www.khanacademy.org/math/precalculus/precalc-matrices/determinant-of-2x2-matrix/e/matrix_determinant

<http://www.purplemath.com/modules/determs.htm>

<http://www.coolmath.com/algebra/14-determinants-cramers-rule/01-determinants-cramers-rule-2x2-02>

https://youtu.be/OU9sWHk_dlw

<https://youtu.be/0S5VfMNqgaM>

<https://youtu.be/Ympm-AxmJ14>

Homework Assignment

p.588 #33,35,37,41,44,45