

### 7.2 Matrix Algebra

Target 8E: Represent a system of linear equations as a single matrix equation in a vector variable  
Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

#### Review of Prior Concepts

Find the value of the variables.

1.  $\begin{bmatrix} 3 \\ 2k \end{bmatrix} + \begin{bmatrix} m \\ -4 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$

$3+m=5$        $2k-4=11$   
 $m=2$                $2k=15$   
 $k=7.5$

2.  $\begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}$

$-2+a=-2$        $1+b=1$   
 $a=0$                $b=0$   
 $3+c=3$            $7+d=7$   
 $c=0$                $d=0$

3.  $\begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}$

$-2a+1c=-2$        $-2b+d=1$   
 $3a+7c=3$            $3b+7d=7$   
 $c=0$                $b=0$   
 $a=1$                $d=1$

### More Practice

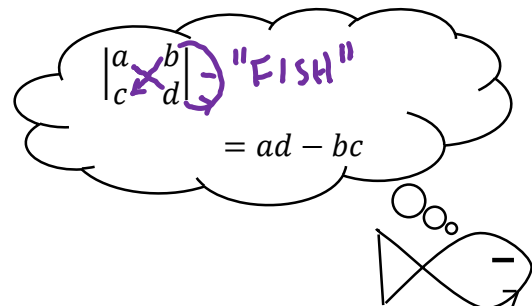
#### Solving Equal Matrices

- <http://www.purplemath.com/modules/matrices3.htm>
- <http://www.onlinemathlearning.com/equal-matrices.html>
- <http://math.tutorvista.com/algebra/equal-matrices.html>
- <https://www.youtube.com/watch?v=nOIRFQsHGOg>
- <https://www.youtube.com/watch?v=LBfv7LtZHWk>

### Determinant of a $2 \times 2$ Matrix

Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then the determinant of A is:

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



Examples:

1. Find  $\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}$

$\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = 3(-1) - 2(1)$   
 $= -3 - 2$   
 $= -5$

2. Find  $\det A$ , where  $A = \begin{bmatrix} 6 & 3 \\ 1 & -2 \end{bmatrix}$

$\det A = \begin{vmatrix} 6 & 3 \\ 1 & -2 \end{vmatrix}$   
 $= 6(-2) - 3(1)$   
 $= -12 - 3$   
 $= -15$

3. Using your graphing calculator, find  $\begin{vmatrix} 3 & -1 & 7 \\ 2 & 0 & 1 \\ 1 & 4 & 2 \end{vmatrix}$ .

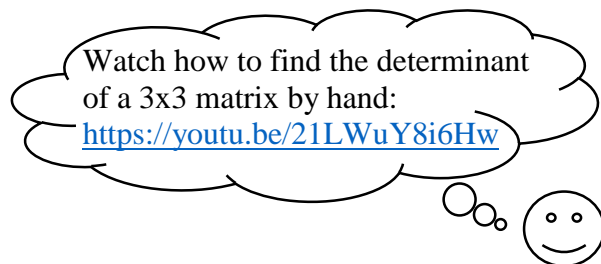
menu

MATRIX  
VECTOR

$\begin{vmatrix} 3 & -1 & 7 \\ 2 & 0 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 47$

Watch how to find the determinant of a  $3 \times 3$  matrix by hand:

<https://youtu.be/21LWuY8i6Hw>

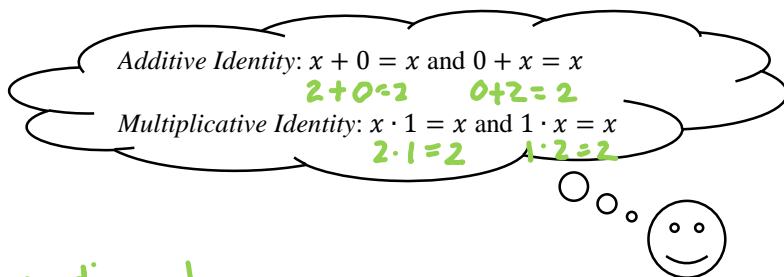


**Identity Matrix** (for a square matrix)

$$A \cdot I_n = I_n \cdot A = A$$

where  $I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$

→ ones along diagonal



Examples:

1. Write  $I_2$

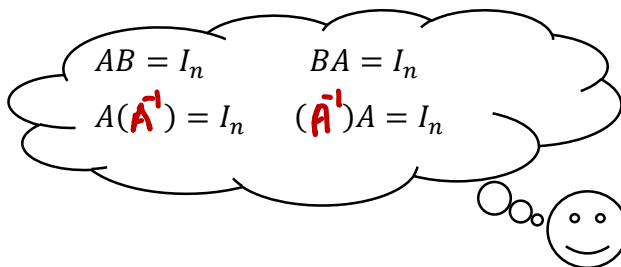
$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2. Write  $I_4$

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Inverse of a Square Matrix**

If  $A$  and  $B$  are both  $n \times n$  matrices and  $AB = I_n$  and  $BA = I_n$ , then  $B$  is the inverse of  $A$  ( $B$  is  $A^{-1}$ ).



\*Matrix  $A$  has an inverse if  $\det A \neq 0$

**Find the Inverse of a  $2 \times 2$  Matrix**

If  $\det A \neq 0$  and  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , then

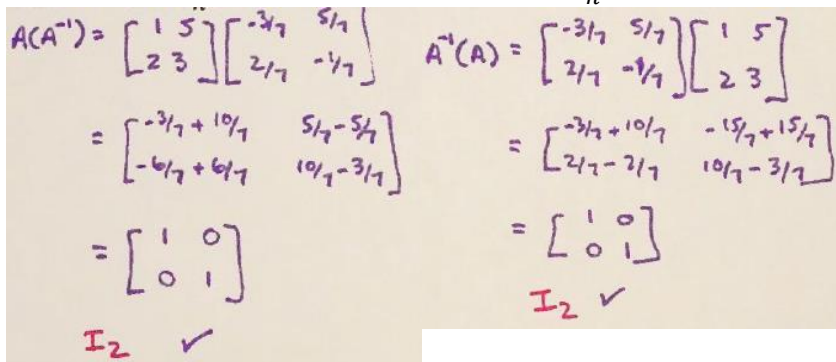
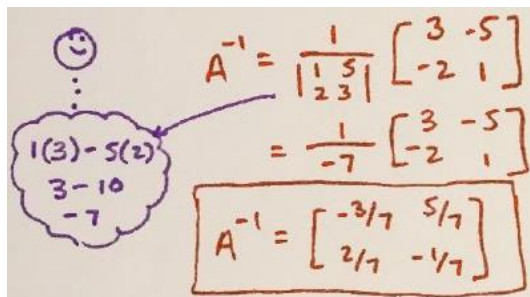
$$ad - bc \neq 0$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples:

1. Find the inverse of  $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$  and check your answer.

Check:  $AB = BA = I_n$ ?



2.  $A = \begin{bmatrix} 6 & 3 \\ 10 & 5 \end{bmatrix}$ . Find  $A^{-1}$ .

$A^{-1} = \frac{1}{\begin{vmatrix} 6 & 3 \\ 10 & 5 \end{vmatrix}} \begin{bmatrix} 5 & -3 \\ -10 & 6 \end{bmatrix}$   
 $= \frac{1}{0} \begin{bmatrix} 5 & -3 \\ -10 & 6 \end{bmatrix}$   
 no inverse

3. Find inverse of:  $\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & -2 \\ 0 & 1 & 1 \end{bmatrix} = A$

or

$A^{-1} = \begin{bmatrix} -2/3 & -1/3 & 4/3 \\ 1/6 & 1/3 & 1/6 \\ -1/6 & -1/3 & 5/6 \end{bmatrix}$

Watch how to find the inverse of a 3x3 matrix by hand:

[https://youtu.be/YvjkPF6C\\_LI](https://youtu.be/YvjkPF6C_LI)

### More Practice

#### Inverse Matrices

<https://www.mathsisfun.com/algebra/matrix-inverse.html>

[https://www.khanacademy.org/math/precalculus/precalc-matrices/practice-finding-inverses-of-2x2-matrices/e/matrix\\_inverse\\_2x2](https://www.khanacademy.org/math/precalculus/precalc-matrices/practice-finding-inverses-of-2x2-matrices/e/matrix_inverse_2x2)

<http://www.intmath.com/matrices-determinants/5-inverse-matrix.php>

[https://youtu.be/OU9sWHk\\_dlw](https://youtu.be/OU9sWHk_dlw)

[https://youtu.be/y4B\\_EC5MNS8](https://youtu.be/y4B_EC5MNS8)

#### Determinants

<http://www.mathsisfun.com/algebra/matrix-determinant.html>

<http://www.virtualnerd.com/algebra-2/matrices/determinants-cramers-rule/determinants/determinant-2-by-2>

[https://www.khanacademy.org/math/precalculus/precalc-matrices/determinant-of-2x2-matrix/e/matrix\\_determinant](https://www.khanacademy.org/math/precalculus/precalc-matrices/determinant-of-2x2-matrix/e/matrix_determinant)

<http://www.purplemath.com/modules/determs.htm>

<http://www.coolmath.com/algebra/14-determinants-cramers-rule/01-determinants-cramers-rule-2x2-02>

[https://youtu.be/OU9sWHk\\_dlw](https://youtu.be/OU9sWHk_dlw)

<https://youtu.be/0S5VfMNqgaM>

<https://youtu.be/Ympm-AxmJ14>

### Homework Assignment

p.588 #33,35,37,41,44,45