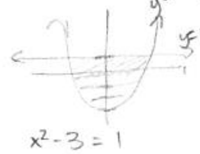
MULTIPLE CHOICE

Choose the answer that gives the area of the region whose boundaries are given.

1. The parabola $y = x^2 - 3$ and the line $y = 1$

a) 8/3
b) 32
c) 32/3
d) 16/3
e) none of these

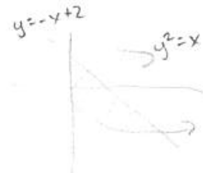


$x^2 - 3 = 1$
$x^2 - 4 = 0$
$(x-2)(x+2) = 0$
$x = -2, x = 2$

$$\text{Area} = \int_{-2}^{2} (1 - (x^2-3))\,dx = 2\left[\int_0^2 (1-(x^2-3))\,dx\right]$$
$$= 2\left[\int_0^2 -x^2+4\,dx\right]$$
$$= 2\left(-\tfrac{1}{3}x^3 + 4x\right)\Big|_0^2$$
$$= 2\left[\left(-\tfrac{8}{3}+8\right) - 0\right] \Rightarrow 2\left(-\tfrac{8}{3}+\tfrac{24}{3}\right)$$
$$= 2\left(\tfrac{16}{3}\right) = \boxed{32/3}$$

2. The parabola $y^2 = x$ and the line $x + y = 2$

a) 5/2
b) 3/2
c) 11/6
d) 9/2
e) 29/6



$x = y^2 \quad x = -y + 2$
$y^2 = -y + 2$
$y^2 + y - 2 = 0$
$y = -2, y = 1$

$$\text{Area} = \int_{-2}^{1} (-y+2) - y^2\,dy$$
$$= \int_{-2}^{1} (-y^2 - y + 2)\,dy$$
$$= -\tfrac{1}{3}y^3 - \tfrac{1}{2}y^2 + 2y\,\Big|_{-2}^{1}$$
$$= \left(-\tfrac{1}{3} - \tfrac{1}{2} + 2\right) - \left(\tfrac{8}{3} - 2 - 4\right)$$
$$= -\tfrac{1}{3} - \tfrac{1}{2} + 2 - \tfrac{8}{3} + 2 + 4$$
$$= -\tfrac{9}{3} + 8 - \tfrac{1}{2} = \boxed{\tfrac{9}{2}}$$

3. The curve of $y = 2/x$ and $x + y = 3$

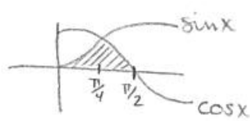
a) $\frac{1}{2} - 2 \ln 2$
b) 3/2
c) $\frac{1}{2} - \ln 4$
d) 5/2
e) $3/2 - \ln 4$

$\frac{2}{x} = 3 - x$
$2 = 3x - x^2$
$x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$x = 1, x = 2$

$$\text{Area} = \int_1^2 \left(3 - x - \tfrac{2}{x}\right)dx$$
$$= \left(3x - \tfrac{1}{2}x^2 - 2\ln|x|\right)\Big|_1^2$$
$$= 6 - 2 - \ln 2 - \left(3 - \tfrac{1}{2} - 0\right)$$
$$= 4 - \ln 2 - 3 + \tfrac{1}{2} = \boxed{\tfrac{3}{2} - \ln 2}$$

4. In the 1st quadrant, bounded below by the x-axis and above by the curves of $y = \sin x$ and $y = \cos x$.

a) $2 - \sqrt{2}$
b) $2 + \sqrt{2}$
c) 2
d) $\sqrt{2}$
e) $2\sqrt{2}$

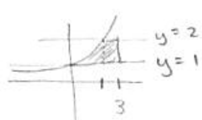


$\sin x = \cos x$
$x = \pi/4$ (Quad I)

$$\text{Area} = \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$
$$= -\cos x\Big|_0^{\pi/4} + \sin x\Big|_{\pi/4}^{\pi/2}$$
$$= -\cos\pi/4 - (-\cos 0) + \sin\pi/2 - \sin\pi/4$$
$$= -\sqrt{2}/2 + 1 + 1 - \sqrt{2}/2 \longrightarrow \boxed{-\sqrt{2} + 2}$$

5. The area bounded by $y = e^x$, $y = 1$, $y = 2$, and $x = 3$ is equal to

a) $3 + \ln 2$
b) $3 - 3 \ln 3$
c) $4 + \ln 2$
d) $3 - \frac{1}{2} \ln^2 2$
e) $4 - \ln 4$



$e^x = 2$
$x = \ln 2$

$$\text{Area} = \int_0^{\ln 2} (e^x - 1)\,dx + \int_{\ln 2}^{3} (2-1)\,dx$$
$$= (e^x - x)\Big|_0^{\ln 2} + (x)\Big|_{\ln 2}^{3}$$
$$= e^{\ln 2} - \ln 2 - (e^0 - 0) + 3 - \ln 2$$
$$= 2 - \ln 2 - 1 + 3 - \ln 2$$
$$= 4 - 2\ln 2 \longrightarrow 4 - \ln 2^2 = \boxed{4 - \ln 4}$$