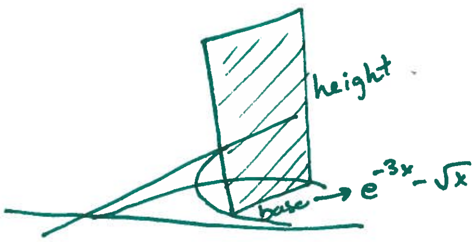
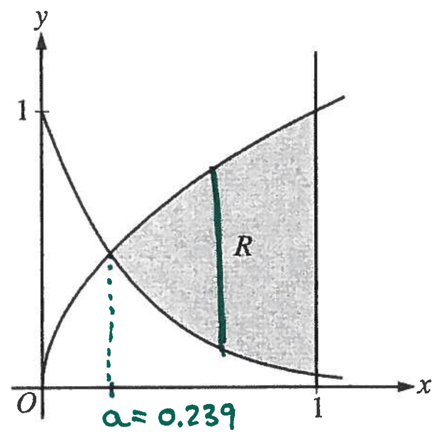
A graphing calculator is required for some problems or parts of problems.





1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

→ need Area formula in terms of x

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

$h = 5(\text{base})$

Area rectangle

$A = bh$

$= b(5b)$

$= 5b^2$

$= 5(e^{-3x} - \sqrt{x})^2$

Volume $= \int_a^1 5(e^{-3x} - \sqrt{x})^2\, dx$

$= \cancel{0.3443}$

$= 1.554$

---

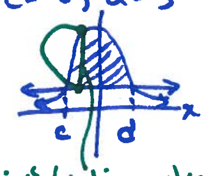
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

Area $= \dfrac{\pi r^2}{2}$

Area $= \dfrac{\pi}{2}\left[\dfrac{1}{2}\left(\dfrac{20}{1+x^2} - 2\right)\right]^2$

$c = -3,\ d = 3$



radius $= \frac{1}{2}$(diameter)

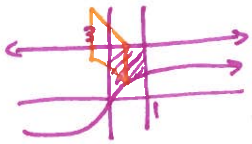
$= \frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)$

Volume $= \int_c^d \frac{\pi}{2}\left(\frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)\right)^2 dx$

$= 174.268$

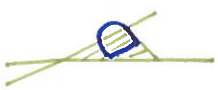
86. The base of a solid is the region in the first quadrant bounded by the y-axis, the graph of $y = \tan^{-1} x$, the horizontal line $y = 3$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

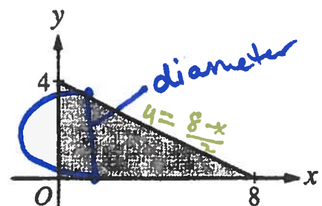
(A) 2.561 $\quad$ $\boxed{\text{(B)}}$ 6.612 $\quad$ (C) 8.046 $\quad$ (D) 8.755 $\quad$ (E) 20.773



square
Area $=(\text{side})^2$
$=(3-\tan^{-1}x)^2$

Volume $= \int_0^1 (3-\tan^{-1}x)^2\,dx$
$= 6.612$





radius $= \frac{1}{2}(\text{diameter})$
$= \frac{1}{2}\left(\frac{8-x}{2}\right)$

86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

$x+2y=8$
$2y=8-x$
$y=\frac{8-x}{2}$

(A) 12.566 $\quad$ (B) 14.661 $\quad$ $\boxed{\text{(C)}}$ 16.755 $\quad$ (D) 67.021 $\quad$ (E) 134.041

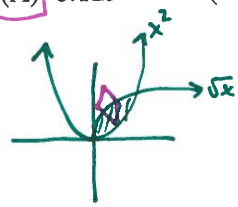
semicircle
Area $= \frac{\pi}{2}(r)^2$
$= \frac{\pi}{2}\left(\frac{1}{2}\left(\frac{8-x}{2}\right)\right)^2$

Volume $= \int_0^8 \frac{\pi}{2}\left(\frac{1}{2}\left(\frac{8-x}{2}\right)\right)^2 dx$
$= 16.755$

---

92. Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph of $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x-axis are squares. What is the volume of the solid?

$\boxed{\text{(A)}}$ 0.129 $\quad$ (B) 0.300 $\quad$ (C) 0.333 $\quad$ (D) 0.700 $\quad$ (E) 1.271



square
Area $= (\text{side})^2$
$= (\sqrt{x}-x^2)^2$

Volume $= \int_0^1 (\sqrt{x}-x^2)^2\,dx$
$= 0.129$