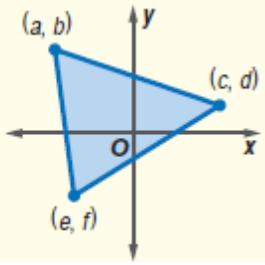
7.3 Solve Systems of Equations Using Matrices

Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Review of Prior Concepts

The area of a triangle having vertices at (a, b), (c, d), and (e, f) is $|A|$, where

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$



Find the area of a triangle whose vertices are: $(-2,1)$, $(3,7)$ and $(8,0)$.

More Practice

Area of a Triangle given Vertices

http://www.mathplanet.com/education/algebra-2/matrices/determinants
http://www.purplemath.com/modules/detprobs.htm
https://www.youtube.com/watch?v=bkJX3q7wvJc

SAT Connection

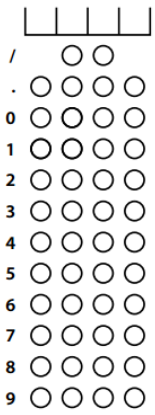
Heart of Algebra

6. Algebraically solve systems of two linear equations in two variables

Example:

$$x + y = -9$$
$$x + 2y = -25$$

According to the system of equations above, what is the value of x ?

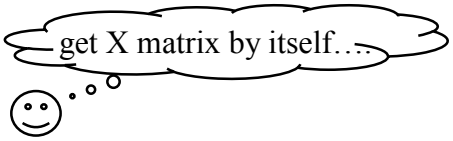


NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Solving System of Equations Using Inverse Matrices

If $AX = B$, where A, B, and X are matrices, then



$$AX = B$$

$$AX = \quad B$$

$$X = \quad B$$

(if A^{-1} exists)

Examples:

1. Solve the system of equations: $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$

2. Solve the system of equations: $\begin{cases} x - y + 2z = -3 \\ 2x + y - z = 0 \\ -x + 2y - 3z = 7 \end{cases}$

3. Find x and y if $BX = A$, where $A = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

More Practice

Solving Systems Using Inverse Matrices

http://www.mathplanet.com/education/algebra-2/matrices/using-matrices-when-solving-system-of-equations
http://math.uww.edu/~mcfarlat/matrix.htm
https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html
https://youtu.be/Re1F4d24Fxc
https://youtu.be/0_DYEFtlCiM
https://youtu.be/FlLsxlWD6a8

Homework Assignment

p.602 #25,49,51,53,55,67,69 (answer all questions using inverse Matrices methods)

SAT Connection

Solution

The correct answer is 7. Subtracting the left and right sides of $x + y = -9$ from the corresponding sides of $x + 2y = -25$ gives $(x + 2y) - (x + y) = -25 - (-9)$, which is equivalent to $y = -16$. Substituting -16 for y in $x + y = -9$ gives $x + (-16) = -9$, which is equivalent to $x = -9 - (-16) = 7$.