DATE: _____

Unit 8 (Chapter 6 & 7): Matrics & Vectors **Pre-Calculus 2016-2017**

7.3 Solve Systems of Equations Using Matrices

Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

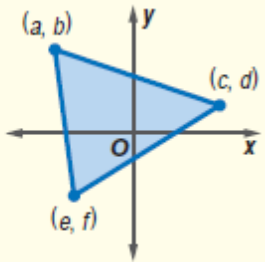
Review of Prior Concepts

The area of a triangle having vertices at (a, b), (c, d), and (e, f) is $|A|$, where

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$

absolute value

determinant



Find the area of a triangle whose vertices are: $(-2,1)$, $(3,7)$ and $(8,0)$.

(a,b) (c,d) (e,f)

$$A = \frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 3 & 7 & 1 \\ 8 & 0 & 1 \end{vmatrix} = 32.5$$

More Practice

Area of a Triangle given Vertices

http://www.mathplanet.com/education/algebra-2/matrices/determinants

http://www.purplemath.com/modules/detprobs.htm

https://www.youtube.com/watch?v=bkJX3q7wvJc

SAT Connection

Heart of Algebra

6. Algebraically solve systems of two linear equations in two variables

Example:

$$x + y = -9$$
$$x + 2y = -25$$

According to the system of equations above, what is the value of x ?

By substitution,

$x+y=-9$

$y=-9-x$

$x+2y=-25$

$x+2(-9-x)=-25$

$x-18-2x=-25$

$-x-18=-25$

$-x=-7$

$x=7$

By linear combination,

$x+y=-9$ (-2) $\rightarrow$ $-2x-2y=18$

$x+2y=-25$ $\rightarrow$ $x+2y=-25$

$-x=-7$

$x=7$

	7			
/	○	○		
.	○	○	○	○
0	○	○	○	○
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	●	○	○	○
8	○	○	○	○
9	○	○	○	○

NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Solving System of Equations Using Inverse Matrices

If $AX = B$, where A, B, and X are matrices, then

get X matrix by itself…

$$AX = B$$

$$(A^{-1})AX = A^{-1}B$$

I, identity matrix

$$X = A^{-1}B \quad \text{(if } A^{-1} \text{ exists)}$$

Examples:

1. Solve the system of equations: $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$

$$\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3(1)-(-2)(-1)}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Remember $(2\times 2)(2\times 1)$, 2×1

$$= \begin{bmatrix} 1(0)+2(5) \\ 1(0)+3(5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$x = 10$, $y = 15$ or $(10, 15)$

2. Solve the system of equations: $\begin{cases} x - y + 2z = -3 \\ 2x + y - z = 0 \\ -x + 2y - 3z = 7 \end{cases}$

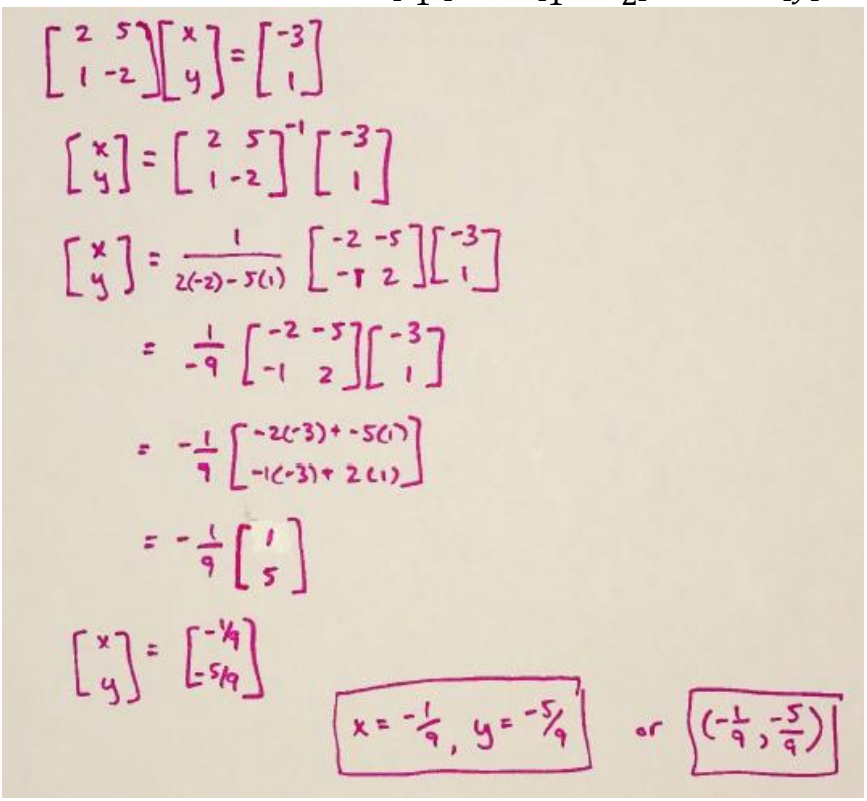
$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix}^{-1}\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 3 \end{bmatrix}$$

$x = -2$, $y = 7$, $z = 3$ or $(-2, 7, 3)$

3. Find x and y if $BX = A$, where $A = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$.



More Practice

Solving Systems Using Inverse Matrices

http://www.mathplanet.com/education/algebra-2/matrices/using-matrices-when-solving-system-of-equations
http://math.uww.edu/~mcfarlat/matrix.htm
https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html
https://youtu.be/Re1F4d24Fxc
https://youtu.be/0_DYEFtlCiM

Homework Assignment

p.602 #25,49,51,53,55,67,69 (answer all questions using inverse Matrices methods)

SAT Connection

Solution

The correct answer is 7. Subtracting the left and right sides of $x + y = -9$ from the corresponding sides of $x + 2y = -25$ gives $(x + 2y) - (x + y) = -25 - (-9)$, which is equivalent to $y = -16$. Substituting -16 for y in $x + y = -9$ gives $x + (-16) = -9$, which is equivalent to $x = -9 - (-16) = 7$.