Volume Using Cross Sections

A cross section is the **shape** we get when cutting straight through an object, a view into the **inside** made by cutting through the object.

Examples:

Tree





Visualizing Volume using the Area of a Cross Section

Example:

• Sketch the area between the graphs of

$$y = -\frac{1}{12}x^3 + \frac{1}{6}x^2 + \frac{11}{12}x - 1 \text{ and}$$

$$y = \frac{1}{12}x^3 - \frac{1}{6}x^2 - \frac{11}{12}x + 1 \text{ on the interval [1,4]}.$$

• Use this area as the base of a 3-dimensional solid where the cross sections are semi-circles.

• Sketch a 3-D image of this solid.

Finding the Volume using the Area of a Cross Section

Example:

Let R be the region bounded by the graphs of

$$y = -\frac{1}{12}x^3 + \frac{1}{6}x^2 + \frac{11}{12}x - 1$$
 and

$$y = \frac{1}{12}x^3 - \frac{1}{6}x^2 - \frac{11}{12}x + 1$$
 on the interval [1,4].

Find the volume of the solid that has R as its base if every cross section by a plane perpendicular to the x-axis are semi-circles.

Volume Using Cross Sections:

① Identify the shape of the cross section

② Find the area of that cross section

③ Sum up the total # of cross sections to get the volume