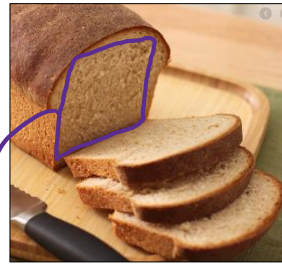## Volume Using Cross Sections

A cross section is the **shape** we get when cutting straight through an object, a view into the **inside** made by cutting through the object.
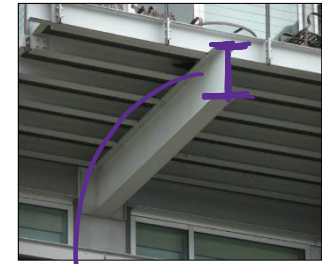
Examples:

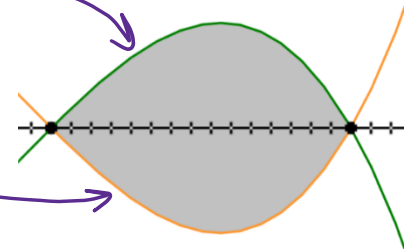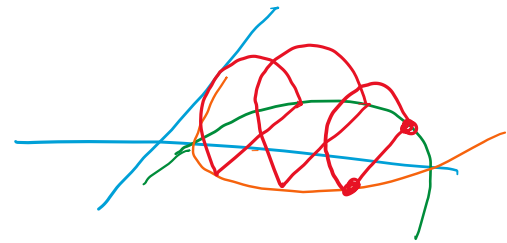| Tree	Bread	I-Beam

circle   square(ish)   "I"

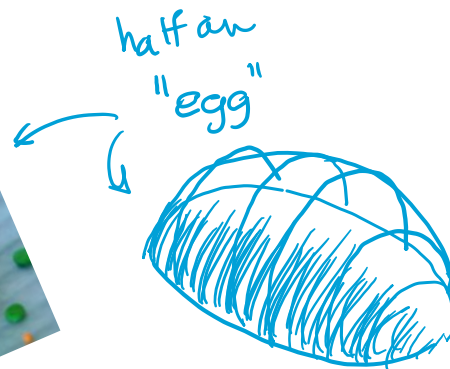### Visualizing Volume using the Area of a Cross Section

Example:

- Sketch the area between the graphs of
  $y = -\frac{1}{12}x^3 + \frac{1}{6}x^2 + \frac{11}{12}x - 1$ and
  $y = \frac{1}{12}x^3 - \frac{1}{6}x^2 - \frac{11}{12}x + 1$ on the interval [1,4].

- Use this area as the base of a 3-dimensional solid where the cross sections are semi-circles.

semicircle

- Sketch a 3-D image of this solid.

half an "egg"

Example:

Let R be the region bounded by the graphs of

$f(x)$ $y = -\frac{1}{12}x^3 + \frac{1}{6}x^2 + \frac{11}{12}x - 1$ and
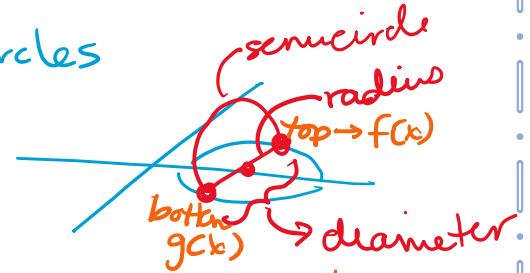
$g(x)$ $y = \frac{1}{12}x^3 - \frac{1}{6}x^2 - \frac{11}{12}x + 1$ on the interval $[1,4]$.

Find the volume of the solid that has R as its base if every cross section by a plane perpendicular to the x-axis are semi-circles.

Volume Using Cross Sections:

① Identify the shape of the cross section

① semi-circles

semicircle
radius
top → $f(x)$
bottom $g(x)$ → diameter

② Find the area of that cross section

Area = ?

② Area of semicircle

$A = \frac{1}{2}\pi r^2$

diameter = $f(x) - g(x)$

radius = $\frac{f(x) - g(x)}{2}$

$A = \frac{1}{2}\pi \left(\frac{-\frac{1}{12}x^3 + \frac{1}{6}x^2 + \frac{11}{12}x - 1 - \left(\frac{1}{12}x^3 - \frac{1}{6}x^2 - \frac{11}{12}x + 1\right)}{2} \right)^2$

$= \frac{1}{2}\pi \left(\frac{-\frac{1}{6}x^3 + \frac{1}{3}x^2 + \frac{11}{6}x - 2}{2} \right)^2$

③ Sum up the total # of cross sections to get the volume

$\text{Volume} = \lim_{n \to \infty} \sum_{k=1}^{n} \left(\begin{array}{c} \text{area of} \\ \text{cross section} \end{array} \right)$

$= \int_a^b \left(\begin{array}{c} \text{area of} \\ \text{cross section} \end{array} \right)$

③ $\text{Volume} = \int_1^4 \frac{1}{2}\pi \left(\frac{-\frac{1}{6}x^3 + \frac{1}{3}x^2 + \frac{11}{6}x - 2}{2} \right)^2 dx$

$= \boxed{2.701}$