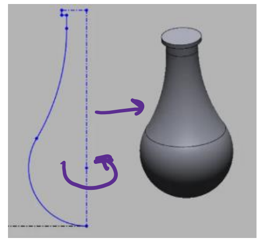
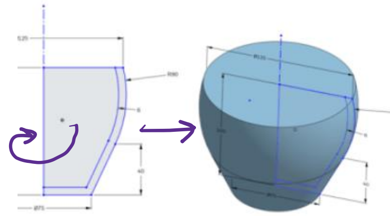
DATE: ___________________

Volume Using Disk Method

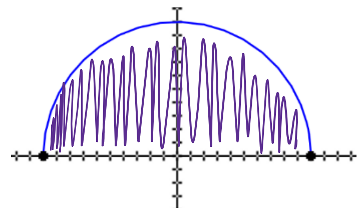
Volume formed by revolving an area around an axis.



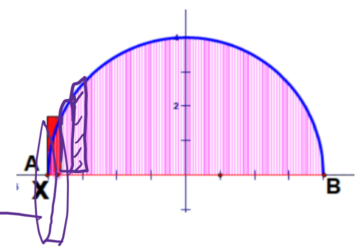



Finding the Volume of a Solid Using the Disk Method

Skètch the area between $y = \sqrt{4 - x^2}$ and the x-axis.



Recall that area under the curve was adding rectangles (Riemann)

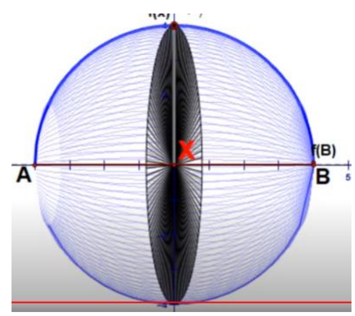


But we need volume,

so take the rectangles and spin around the x-axis.

disk (circle)

Area of circle (disk) $= \pi r^2$



Add up the disks to get… an orange! volume of orange

$$\text{Volume} = \sum_{k=1}^{n} \pi r^2$$

An infinite # of disks…

$$\text{Volume} = \lim_{n \to \infty} \sum_{k=1}^{n} \pi r^2$$

$$= \int_a^b \pi (r(x))^2 \, dx$$

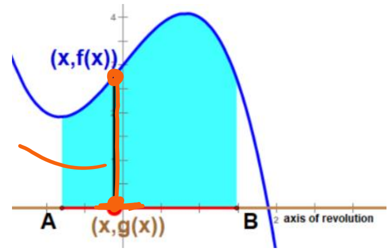
Volume of a Solid Disk Method

Revolve around an x-axis (or a horizontal axis)

$$\text{Volume} = \pi \int_a^b \underbrace{(R(x))}_{\text{radius of the disk}}{}^2 \; dx$$

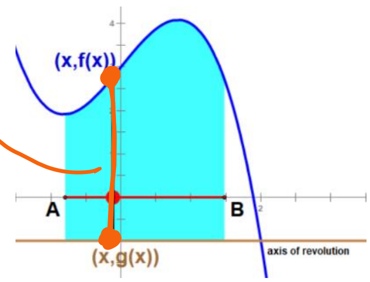
ex: $\text{Volume} = \pi \int_A^B (f(x) - 0)^2 \, dx$

radius (rectangle)



ex: $\text{Volume} = \pi \int_A^B (f(x) - g(x))^2 \, dx$

radius (rectangle)

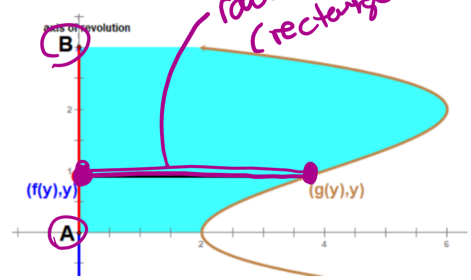


Revolve around an y-axis (or a vertical axis)

$$\text{Volume} = \pi \int_c^d (R(y))^2 \; dy$$

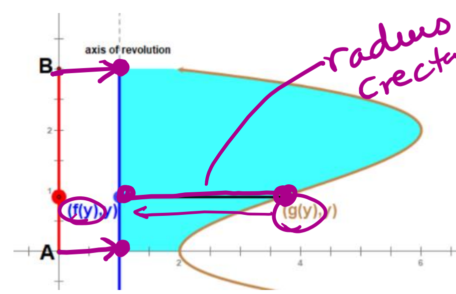
ex: $\text{Volume} = \pi \int_A^B (g(y) - 0)^2 \, dy$

radius (rectangle)



ex: $\text{Volume} = \pi \int_A^B (g(y) - f(y))^2 \, dy$

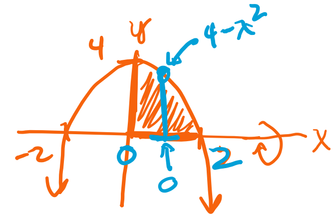
radius (rectangle)



Example 1:

Find the volume of the solid formed by rotating the region in Quadrant I bounded by $y = 4 - x^2$, the x-axis, and the y-axis about the x-axis.

$$\text{Volume} = \pi \int_a^b (R(x))^2\, dx$$



$$\text{Volume} = \pi \int_0^2 (4-x^2-0)^2\, dx$$

$$= \pi \int_0^2 (4-x^2)^2\, dx$$

$$= \pi \int_0^2 (16-8x^2+x^4)\, dx$$

$$= \pi \left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2$$

$$= \pi \left(32 - \frac{64}{3} + \frac{32}{5} - 0\right)$$

$$= 32\pi \left(1 - \frac{2}{3} + \frac{1}{5}\right)$$

$$= 32\pi \left(\frac{15}{15} - \frac{10}{15} + \frac{3}{15}\right)$$

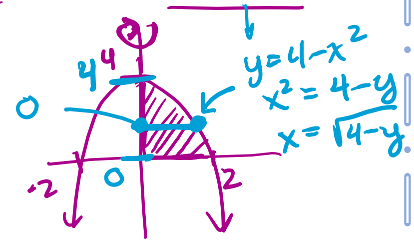
$$= 32\pi \left(\frac{8}{15}\right)$$

$$= \boxed{\frac{256}{15}\pi}$$

Example 2:

Find the volume of the solid formed by rotating the region in Quadrant I bounded by $y = 4 - x^2$, the x-axis, and the y-axis about the y-axis.

$$\text{Volume} = \pi \int_c^d (R(y))^2\, dy$$



$$\text{Volume} = \pi \int_0^4 \left(\sqrt{4-y} - 0\right)^2 dy$$

$$= \pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy$$

$$= \pi \int_0^4 (4-y)\, dy$$

$$= \pi \left(4y - \frac{1}{2}y^2\right)\Big|_0^4$$

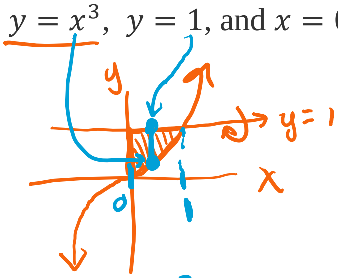
$$= \pi (16 - 8) = \boxed{8\pi}$$

Example 3:

Find the volume of the solid formed by rotating the region bounded by $y = x^3$, $y = 1$, and $x = 0$ about the line $y = 1$.

horizontal line (like an x-axis)

$$\text{Volume} = \pi \int_a^b \left(R(x)\right)^2 dx$$



$x^3 = 1$

$x = \sqrt[3]{1}$

$x = 1$

$$\text{Volume} = \pi \int_0^1 \left(1 - x^3\right)^2 dx$$

$$= \pi \int_0^1 \left(1 - 2x^3 + x^6\right) dx$$

$$= \pi \left(x - \tfrac{1}{2}x^4 + \tfrac{1}{7}x^7 \right)\Big|_0^1$$

$$= \pi \left(1 - \tfrac{1}{2} + \tfrac{1}{7} \right) = \pi \left(\tfrac{14}{14} - \tfrac{7}{14} + \tfrac{2}{14} \right)$$

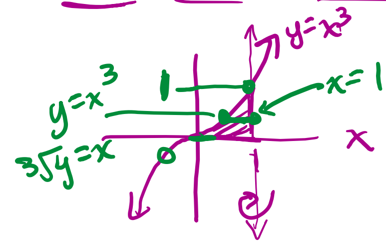
$$= \boxed{\tfrac{9}{14}\pi}$$

Example 4:

Find the volume of the solid formed by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 1$ about the line $x = 1$.

vertical line (like y-axis)

$$\text{Volume} = \pi \int_c^d \left(R(y)\right)^2 dy$$



$y = x^3$

$y = 1^3$

$y = 1$

$$\text{Volume} = \pi \int_0^1 \left(1 - \sqrt[3]{y}\right)^2 dy$$

$$= \pi \int_0^1 \left(1 - y^{1/3}\right)^2 dy$$

$$= \pi \int_0^1 \left(1 - 2y^{1/3} + y^{2/3}\right) dy$$

$$= \pi \left(y - 2\left(\tfrac{3}{4}y^{4/3}\right) + \tfrac{3}{5}y^{5/3} \right)\Big|_0^1$$

$$= \pi \left(1 - \tfrac{3}{2} + \tfrac{3}{5} \right)$$

$$= \pi \left(\tfrac{10}{10} - \tfrac{15}{10} + \tfrac{6}{10} \right) = \boxed{\tfrac{1}{10}\pi}$$