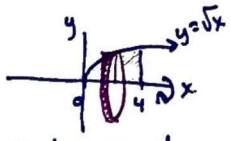
Volume Using the Disk Method

Find the volume of the solid described in each.

1. $y=\sqrt{x}$, $x=0$, $x=4$, revolve about the x-axis



revolve around horizontal axis (work in x's)

Volume $= \pi\int_0^4 (\sqrt{x})^2\,dx$

$= \pi\int_0^4 x\,dx$

$= \pi\left(\frac{1}{2}x^2\right)\Big|_0^4 = \boxed{8\pi}$

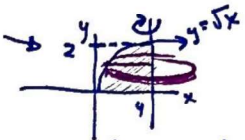
2. $y=\sqrt{x}$, $x=0$, $x=4$, revolve about $x=4$

$y^2 = x$

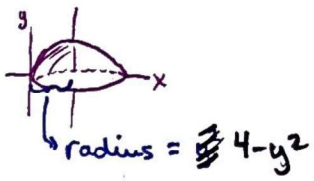
$\sqrt{x} = y$

$\sqrt{4} = y$

$2 = y$



revolve around vertical axis (work in y's)



~~Volume $= \pi\int_0^2 (y^2)^2\,dy$~~

~~$= \pi\int_0^2 y^4\,dy$~~

~~$= \pi\left(\frac{1}{5}y^5\right)\Big|_0^2$~~

~~$= \frac{32}{5}\pi$~~

Volume $= \pi\int_0^2 (4-y^2)^2\,dy$

$= \pi\int_0^2 (16-8y^2+y^4)\,dy$

$= \pi\left(16y-\frac{8}{3}y^3+\frac{1}{5}y^5\right)\Big|_0^2$

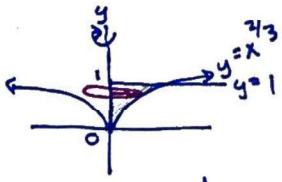
$= \pi\left(32-\frac{64}{3}+\frac{32}{5}-0\right)$

$= \boxed{\frac{256}{15}\pi}$

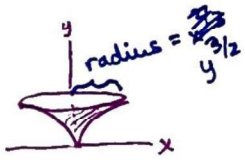
3. $y=x^{2/3}$, $x=0$, $y=1$, revolve about the y-axis

$y^{3/2} = (x^{2/3})^{3/2}$

$y^{3/2} = x$



revolve around vertical axis (work in y's)

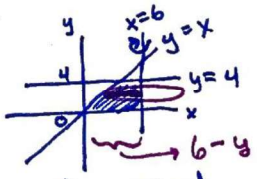


Volume $= \pi\int_0^1 (y^{3/2})^2\,dy$

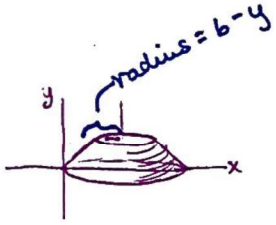
$= \pi\int_0^1 y^3\,dy$

$= \pi\left(\frac{1}{4}y^4\right)\Big|_0^1 = \boxed{\frac{1}{4}\pi}$

4. $y=x$, $y=0$, $y=4$, $x=6$, revolve about $x=6$



revolve around vertical axis (work in y's)



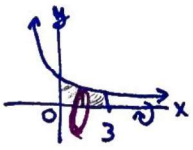
Volume $= \pi\int_0^4 (6-y)^2\,dy$

$= \pi\int_0^4 (36-12y+y^2)\,dy$

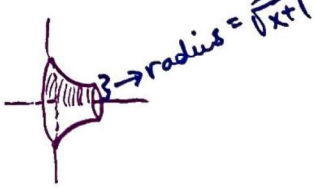
$= \pi\left(36y-6y^2+\frac{1}{3}y^3\right)\Big|_0^4$

$= \pi\left(144-96+\frac{64}{3}\right) = \boxed{\frac{208}{3}\pi}$

5. $y=\dfrac{1}{\sqrt{x+1}}$, $y=0$, $x=0$, $x=3$, revolve about the x-axis



revolve around horizontal axis (work in x's)



Volume $= \pi\int_0^3 \left(\frac{1}{\sqrt{x+1}}\right)^2 dx$

$= \pi\int_0^3 \frac{1}{x+1}\,dx$

$u = x+1$

$du = dx$ ☺

$= \pi \ln|x+1|\,\Big|_0^3$

$= \pi(\ln 4 - \ln 1)$

$= \boxed{\pi \ln 4}$