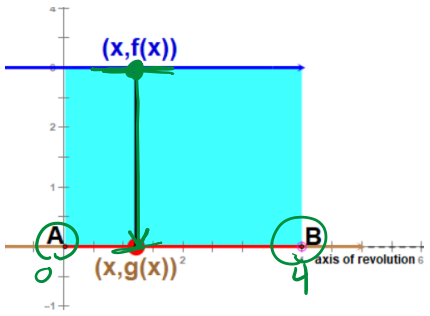
DATE: _____

Volume Using Washer Method

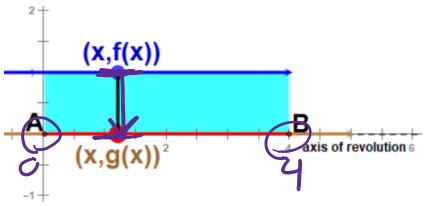
Find the volume of the solid generated by $x = 0$, $x = 4$, $y = 0$, and $y = 3$ revolved about the x-axis.

$$\text{Volume} = \pi \int_0^4 (3-0)^2 \, dx$$
$$= \pi \int_0^4 9 \, dx$$
$$= \pi (9x)\Big|_0^4$$
$$= \pi (36-0) = \boxed{36\pi}$$



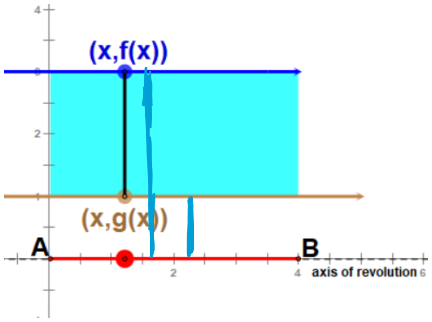
Find the volume of the solid generated by $x = 0$, $x = 4$, $y = 0$, and $y = 1$ revolved about the x-axis.

$$\text{Volume} = \pi \int_0^4 (1-0)^2 \, dx$$
$$= \pi \int_0^4 1 \, dx$$
$$= \pi (x)\Big|_0^4 = \pi(4-0) = \boxed{4\pi}$$



Find the volume of the solid generated by $x = 0$, $x = 4$, $y = 1$, and $y = 3$ revolved about the x-axis.

$$\text{Volume} = 36\pi - 4\pi$$
$$= \boxed{32\pi}$$



Sample Washers:

Hardware

Candy

Vase

Volume of a Solid Washer Method

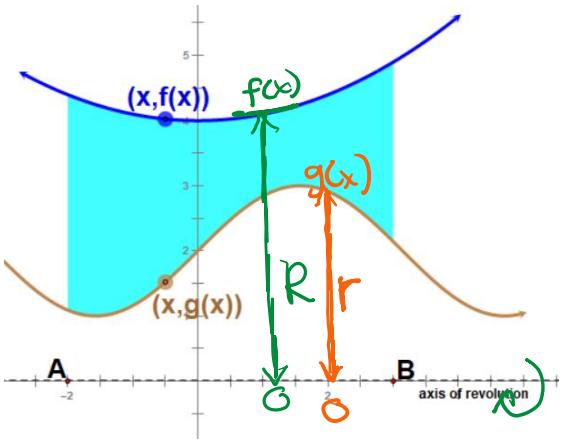
Revolve around an x-axis (or a horizontal axis)

$$\text{Volume} = \pi \int_a^b \left((R(x))^2 - (r(x))^2 \right) dx$$

outer radius — inner radius

ex: $\text{Volume} = \pi \int_A^B \left((f(x)-0)^2 - (g(x)-0)^2 \right) dx$

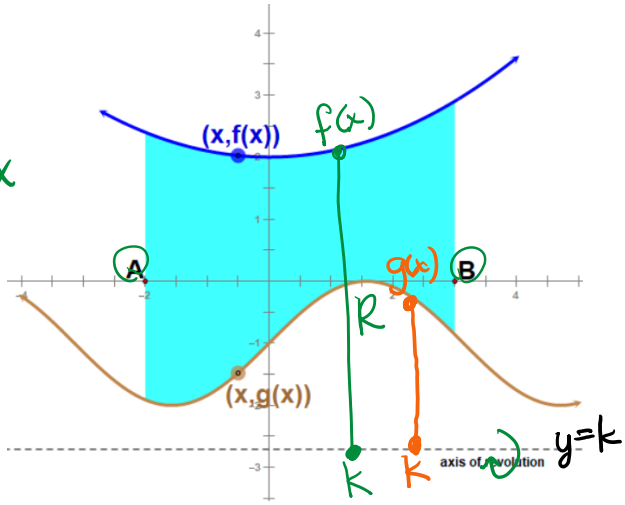
$= \pi \int_A^B \left((f(x))^2 - (g(x))^2 \right) dx$



$\text{Volume} = \pi \int_a^b \left((R(x))^2 - (r(x))^2 \right) dx$

outer radius — inner radius

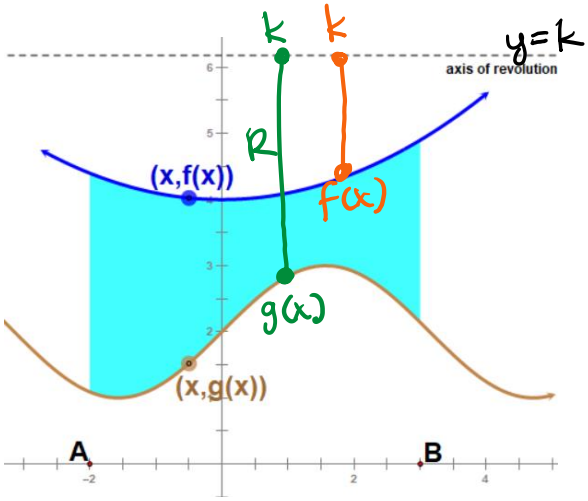
$\text{Volume} = \pi \int_A^B \left((f(x)-k)^2 - (g(x)-k)^2 \right) dx$



$\text{Volume} = \pi \int_a^b \left((R(x))^2 - (r(x))^2 \right) dx$

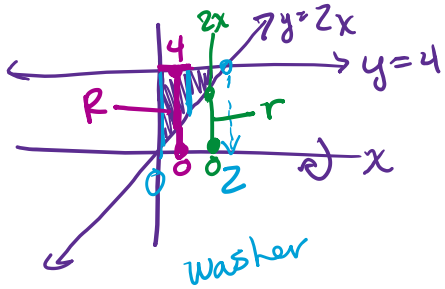
outer radius — inner radius

$\text{Volume} = \pi \int_A^B \left((k-g(x))^2 - (k-f(x))^2 \right) dx$



Example 1:

Find the volume of the solid formed by revolving the area of the region bounded by $y = 2x$, $y = 4$, and $x = 0$ about the x-axis.



$2x = 4$

$x = 2$

$$\text{Volume} = \pi \int_a^b \left((R(x))^2 - (r(x))^2 \right) dx$$

(outer radius, inner radius)

$$= \pi \int_0^2 \left[(4-0)^2 - (2x-0)^2 \right] dx$$

$$= \pi \int_0^2 (4^2 - (2x)^2)\, dx$$

$$= \pi \int_0^2 (16 - 4x^2)\, dx$$

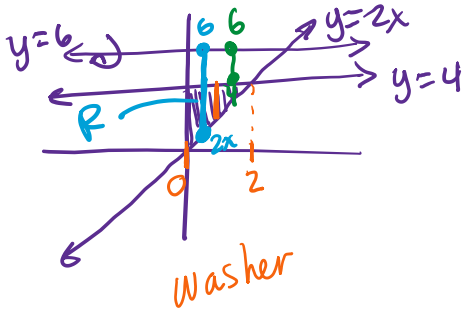
$$= \pi \left(16x - \frac{4}{3}x^3\right)\Big|_0^2$$

$$= \pi \left(32 - \frac{32}{3} - 0\right)$$

$$= 32\pi\left(1 - \frac{1}{3}\right) = 32\pi\left(\frac{2}{3}\right) = \boxed{\frac{64}{3}\pi}$$

Example 2:

Find the volume of the solid formed by revolving the area of the region bounded by $y = 2x$, $y = 4$, and $x = 0$ about the line $y = 6$.



$$\text{Volume} = \pi \int_a^b \left((R(x))^2 - (r(x))^2 \right) dx$$

(outer radius, inner radius)

$$= \pi \int_0^2 \left((6-2x)^2 - (6-4)^2 \right) dx$$

$$= \pi \int_0^2 \left((6-2x)^2 - 2^2 \right) dx$$

$$= \pi \int_0^2 (36 - 24x + 4x^2 - 4)\, dx$$

$$= \pi \int_0^2 (32 - 24x + 4x^2)\, dx$$

$$= \pi \left(32x - 12x^2 + \frac{4}{3}x^3\right)\Big|_0^2$$

$$= \pi \left(64 - 48 + \frac{32}{3} - 0\right)$$

$$= \pi \left(16 + \frac{32}{3}\right)$$

$$= 16\pi\left(1 + \frac{2}{3}\right)$$

$$= 16\pi\left(\frac{5}{3}\right)$$

$$= \boxed{\frac{80}{3}\pi}$$

Volume of a Solid Washer Method

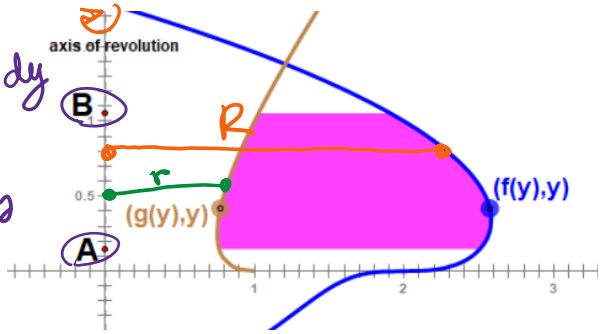
Revolve around an y-axis (or a vertical axis)

$$\text{Volume} = \pi \int_c^d \left((R(y))^2 - (r(y))^2 \right) dy$$

outer radius, inner radius

ex: $\text{Volume} = \pi \int_A^B \left((f(y)-0)^2 - (g(y)-0)^2 \right) dy$

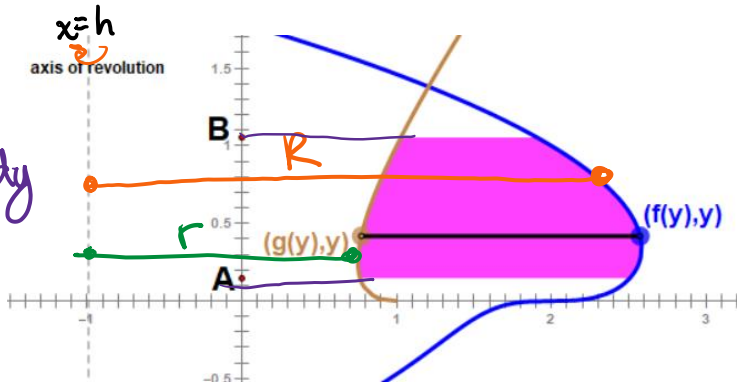
$= \pi \int_A^B \left((f(y))^2 - (g(y))^2 \right) dy$



$\text{Volume} = \pi \int_c^d \left((R(y))^2 - (r(y))^2 \right) dy$

outer radius, inner radius

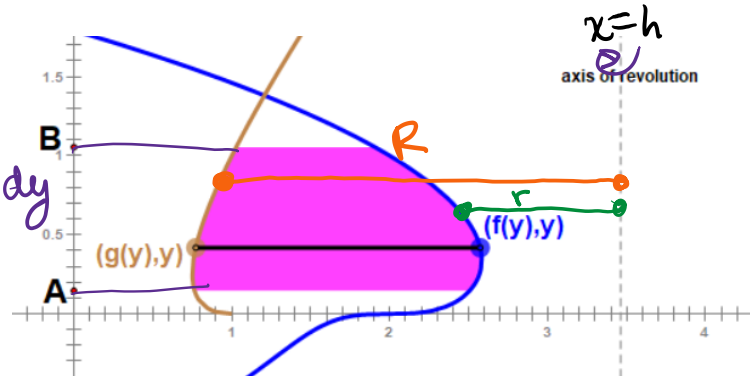
$\text{Volume} = \pi \int_A^B \left((f(y)-h)^2 - (g(y)-h)^2 \right) dy$



$\text{Volume} = \pi \int_c^d \left((R(y))^2 - (r(y))^2 \right) dy$

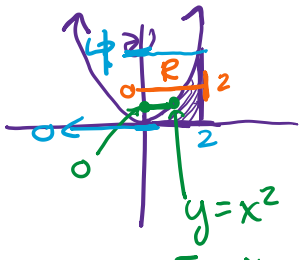
outer radius, inner radius

$\text{Volume} = \pi \int_A^B \left((h-g(y))^2 - (h-f(y))^2 \right) dy$



Example 3:

Find the volume of the solid formed by revolving the area of the region bounded by $y = x^2$, $y = 0$, and $x = 2$ about the y-axis.



in terms of y

$$\text{Volume} = \pi \int_c^d \left((R(y))^2 - (r(y))^2 \right) dy$$

$$= \pi \int_0^4 \left((2-0)^2 - (\sqrt{y}-0)^2 \right) dy$$

$$= \pi \int_0^4 \left(2^2 - (\sqrt{y})^2 \right) dy$$

$$= \pi \int_0^4 (4 - y)\, dy$$

$$= \pi \left(4y - \tfrac{1}{2}y^2 \right) \Big|_0^4$$

$$= \pi (16 - 8 - 0)$$

$$= \boxed{8\pi}$$

$y = x^2$, $x = 2$

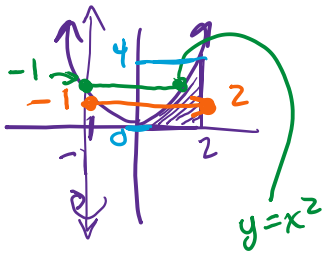
$y = 2^2$

$y = 4$

$\sqrt{y} = x$

Example 4:

Find the volume of the solid formed by revolving the area of the region bounded by $y = x^2$, $y = 0$, and $x = 2$ about the line $x = -1$.



vertical line (like y-axis)

$$\text{Volume} = \pi \int_c^d \left((R(y))^2 - (r(y))^2 \right) dy$$

$$\text{Volume} = \pi \int_0^4 \left((2 - -1)^2 - (\sqrt{y} - -1)^2 \right) dy$$

$$= \pi \int_0^4 \left(3^2 - (\sqrt{y} + 1)^2 \right) dy$$

$$= \pi \int_0^4 \left(9 - (y + 2\sqrt{y} + 1) \right) dy$$

$$= \pi \int_0^4 \left(9 - y - 2y^{1/2} - 1 \right) dy$$

$$= \pi \int_0^4 \left(8 - y - 2y^{1/2} \right) dy$$

$$= \pi \left(8y - \tfrac{1}{2}y^2 - 2\left(\tfrac{2}{3}y^{3/2}\right) \right) \Big|_0^4$$

$$= \pi \left(32 - 8 - \tfrac{4}{3}(\sqrt{4})^3 - 0 \right)$$

$$= \pi \left(24 - \tfrac{32}{3} \right)$$

$$= 8\pi \left(3 - \tfrac{4}{3} \right)$$

$$= \boxed{\tfrac{40}{3}\pi}$$

$y = x^2$, $x = 2$

$y = 2^2$

$y = 4$

$\sqrt{y} = x$