# 7.3 Solve Systems of Equations Using Matrices

Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).

*Review of Prior Concepts*

Find the equation of a parabola that passes through the points $(-1,9)$, $(1,5)$, and $(2,12)$ using Inverse Matrices.

↳ $y = ax^2 + bx + c$

$9 = a(-1)^2 + b(-1) + c \rightarrow a - b + c = 9$

$5 = a(1)^2 + b(1) + c \rightarrow a + b + c = 5$

$12 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 12$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 9 \\ 5 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \quad \boxed{y = 3x^2 - 2x + 4}$$

## More Practice

**Solving Systems Using Inverse Matrices**

http://www.mathplanet.com/education/algebra-2/matrices/using-matrices-when-solving-system-of-equations

http://math.uww.edu/~mcfarlat/matrix.htm

https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html

https://youtu.be/Re1F4d24Fxc

https://youtu.be/0_DYEFtlCiM

https://youtu.be/FlLsxlWD6a8

## Augmented Matrices

**Augmented Matrix** – one matrix with coefficients and answers

*Example:*

Write the system of equations as an augmented matrix: $\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$

$$\left[\begin{array}{ccc:c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array}\right]$$

optional: represents "=" sign

## Solving Augmented Matrices

To solve an augmented matrix,

use Gaussian elimination to have the matrix in **R**educed **R**ow **E**chelon **F**orm.

**R**educed **R**ow **E**chelon **F**orm: $\left[\begin{array}{ccccc:c} 1 & 0 & 0 & \cdots & 0 & a_1 \\ 0 & 1 & 0 & \cdots & 0 & a_2 \\ 0 & 0 & 1 & & & \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & a_n \end{array}\right]$

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*Examples:*

**1.** Solve the system of equations using **R**educed **R**ow **E**chelon **F**orm:

$$\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$$

$$\text{rref}\left(\begin{bmatrix} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{bmatrix}\right) = \begin{array}{c} \begin{matrix} x & y & z & = \end{matrix} \\ \left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \end{array} \begin{matrix} \to x = 2 \\ \to y = -1 \\ \to z = 3 \end{matrix}$$

$$\boxed{(2, -1, 3)}$$

**2.** Solve the system of equations using **R**educed **R**ow **E**chelon **F**orm:

$$\begin{cases} x + y + 3z = 2 \\ 3x + 4y + 10z = 5 \\ x + 2y + 4z = 3 \end{cases}$$

$$\text{rref}\left(\begin{bmatrix} 1 & 1 & 3 & 2 \\ 3 & 4 & 10 & 5 \\ 1 & 2 & 4 & 3 \end{bmatrix}\right) = \begin{array}{c} \begin{matrix} x & y & z & = \end{matrix} \\ \left[\begin{array}{ccc:c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{matrix} \to x + 2z = 0 \\ \to y + z = 0 \\ \to 0 = 1 \end{matrix}$$

$\downarrow$ not true $0 \neq 1$,

$\therefore$, $\boxed{\text{no solution}}$

**3.** Solve the system of equations using **R**educed **R**ow **E**chelon **F**orm:

$$\begin{cases} x - z = 2 \\ -2x + y + 3z = -5 \\ 2x + y - z = 3 \end{cases}$$

notice, no y

$$\text{rref}\left(\begin{bmatrix} 1 & 0 & -1 & 2 \\ -2 & 1 & 3 & -5 \\ 2 & 1 & -1 & 3 \end{bmatrix}\right) = \begin{array}{c} \begin{matrix} x & y & z & = \end{matrix} \\ \left[\begin{array}{ccc:c} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{matrix} \to x - z = 2 \\ \to y + z = -1 \\ \to 0 = 0 \end{matrix}$$

$\downarrow$ always true, zero always equals zero

$\therefore$, infinitely many solutions

where $x - z = 2$ and $y + z = -1$

$x = z + 2$ $\quad$ $y = -z - 1$

$$\boxed{(z+2, -z-1, z)}$$

Now you try…

Solve each system of equations using **R**educed **R**ow **E**chelon **F**orm:

**1.** $\begin{cases} x + 2y + 3z = 9 \\ 2x - y + z = 8 \\ 3x - z = 3 \end{cases}$

$$\text{rref}\left(\begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} \to x = 2 \\ \to y = -1 \\ \to z = 3 \end{matrix}$$

$$\boxed{(2, -1, 3)}$$

not every solution will always be (2,-1,3)… that was a coincidence

**2.** $\begin{cases} x + z = 1 \\ x + y + z = 2 \\ x - y + z = 1 \end{cases}$

$$\text{rref}\left(\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \to x + z = 0 \\ \to y = 0 \\ \to 0 = 1 \end{matrix}$$

$0 \neq 1$

$\therefore$, $\boxed{\text{no solution}}$

**3.** $\begin{cases} 3x + y - 6z = -10 \\ 2x + y - 5z = -8 \\ 6x - 3y + 3z = 0 \end{cases}$

$$\text{rref}\left(\begin{bmatrix} 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \\ 6 & -3 & 3 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to x - z = -2 \\ \to y - 3z = -4 \\ \to 0 = 0 \end{matrix}$$

always true.

$\therefore$, infinite # solutions

where $x - z = -2$   $y - 3z = -4$

$x = +z - 2$   $y = 3z - 4$

$$\boxed{(+z - 2, 3z - 4, z)}$$

## More Practice

**Augmented Matrices**

http://www.purplemath.com/modules/matrices.htm
http://www.mathbootcamps.com/augmented-matrices-and-systems-of-linear-equations/
https://braingenie.ck12.org/skills/106514
https://youtu.be/A_fIRE0NJ8Y
https://youtu.be/dHmqVGXyVG4
https://youtu.be/N9tFrUK83Uk

**Gaussian Elimination for Augmented Matrices**

https://www.mathway.com/examples/linear-algebra/systems-of-linear-equations/solving-using-an-augmented-matrix?id=227
http://tutorial.math.lamar.edu/Classes/Alg/AugmentedMatrix.aspx
https://youtu.be/WiVeiVIu_SM
https://youtu.be/2j5Ic2V7wq4

## Homework Assignment

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